

A Theoretical Framework for R-parity Violation

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Abstract

We propose a theoretical framework for R-parity violation. It is realized by a class of Calabi–Yau compactification of Heterotic string theory. Trilinear R-parity violation in superpotential is either absent or negligibly small without an unbroken symmetry, due to a selection rule based on charge counting of a spontaneously broken $U(1)$ symmetry. Although such a selection rule cannot be applied in general to non-renormalizable operators in the low-energy effective superpotential, it is valid for terms trilinear in low-energy degrees of freedom, and hence can be used as a solution to the dimension-4 proton decay problem in the minimal supersymmetric standard model. Bilinear R-parity violation is generated, but there are good reasons why they are small enough to satisfy its upper bounds from neutrino mass and washout of baryon/lepton asymmetry. All R-parity violating dimension-5 operators can be generated. In this theoretical framework, nucleons can decay through squark-exchange diagrams combining dimension-5 and bilinear R-parity violating operators. $B - L$ breaking neutron decay is predicted.

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1 Introduction

In supersymmetric extensions of the standard model, renormalizable operators

$$W \ni \lambda L \bar{E} L + \lambda' L Q \bar{D} + \lambda'' \bar{D} \bar{U} \bar{D} \sim \mathbf{\bar{5}10\bar{5}} \quad (1.1)$$

break baryon and lepton number symmetries, and hence a proton decays too rapidly. Either the coefficient λ'' of the baryon number violating operator $\bar{D} \bar{U} \bar{D}$ or λ and λ' of the lepton number violating $L \bar{E} L$ and $L Q \bar{D}$ have to be highly suppressed. The most popular solution to

this dimension-4 proton decay problem is to assume an unbroken R parity (or matter parity), which removes the operators in (1.1) altogether.

Various alternative solutions have also been discussed in the literature, some of which are found in a review article [1]. Some solutions assume discrete symmetries other than R-parity (see also [2] and references therein), so that either the first two operators or the last one are(is) forbidden by the discrete symmetry. Although phenomenological consequences of these solutions are quite different from those of R-parity preserving ones, these two classes of solutions share one thing in common, an unbroken discrete symmetry. In Calabi–Yau compactification of Heterotic $E_8 \times E'_8$ string theory, however, discrete symmetries are found only at special points (or subsets) in moduli space. Reasons are not clear why such vacua have to be chosen. References [3, 4], for example, clearly expressed dissatisfaction to solutions relying on unbroken discrete symmetries.

This article presents an alternative solution to the problem without assuming an unbroken symmetry. The essence of the solution is an extra U(1) gauge symmetry with a Fayet–Iliopoulos parameter. The U(1) symmetry is broken spontaneously at high energy,¹ allowing for large Majorana masses of right-handed neutrinos. No unbroken symmetry is left at low energy, but its legacy still remains. There is a selection rule [5] (also known as SUSY-zero mechanism) in how the U(1)-breaking vacuum expectation value (vev) can appear in superpotential of low-energy effective theory, and this rule may be used to make sure that the trilinear R-parity violating couplings (1.1) are absent. In this sense, the solution in this article is certainly along the line of models in [6, 7, 8, 9]. It should be reminded, however, that there are many subtleties in how to use the selection rule in low-energy effective superpotential. It is one of the purposes of section 2 of this article to clarify when the selection rule can be used and when it cannot. This solution fits very well with Calabi–Yau compactification of the Heterotic $E_8 \times E'_8$ string theory (and its dual descriptions) [7, 8]. In such string compactification, moduli fields are not required to be at special points, and U(1)-symmetry breaking vev is not assumed to be hierarchically small in order to make sure that the trilinear couplings (1.1) are sufficiently small. Thus, this solution does not share the unsatisfactory aspect of the solutions with discrete symmetries.

In section 2.2, a class of compactification of the Heterotic string theory is discussed;² technically, it is to assume that a vector bundle has an extension structure, and various low-energy

¹There are also solutions in the literature where such a U(1) symmetry is broken by a hierarchically small expectation value. Such solutions, however, share the same unsatisfactory aspect as those with discrete unbroken symmetries. We need to understand why our vacuum is very close to a U(1)-symmetry enhanced point in the moduli space.

²Partial results of sections 2.2 and 2.5 have been obtained in [7, 8].

degrees of freedom are identified with cohomologies of appropriate sub-bundles. We will see in this framework that holomorphicity controls mixings between *massless* states with different U(1) charges. Thus, the selection rule based on U(1)-charge counting is applied for terms trilinear in massless states, and the absence of R-parity violating terms (1.1) can be guaranteed from the selection rule. 1-loop amplitudes generate a bilinear R-parity violating mass term $W \ni \mu_i L_i H_u$ with μ_i proportional to supersymmetry breaking (SUSY-breaking), and the tree-level contribution can be even smaller. Thus, at the renormalizable level, this framework predicts an R parity violation only in the bilinear terms, which is known not to be terribly bad in phenomenology [1, 10]. We also find that all the dimension-5 operators that violate R parity are generated; the selection rule does not have a predictive power at non-renormalizable level. With a theoretical framework controlling all aspects of R parity violation, we can discuss how key parameters of short-distance description control the coefficients of various R parity violating operators.

Section 3 is devoted to phenomenology that is expected when both bilinear and dimension-5 R parity violating operators exist. Although small trilinear R-parity violating couplings can be generated in the framework of section 2, it turns out that they are so small that they are rarely relevant to phenomenology. Because of negligibly small trilinear R-parity violation, most of phenomenological constraints discussed so far are easily satisfied, considerably simplifying phenomenological study. Remaining constraints from low-energy neutrino masses and washout of baryon/lepton asymmetry are briefly discussed in sections 3.1 and 3.2, respectively. Constraints on R-parity violating decay of the lightest supersymmetry particle (LSP) are reanalyzed in section 3.3, where we exploit the latest understanding of impact of new physics on the Big-Bang Nucleosynthesis (BBN). Section 3.4 is devoted to limits on R-parity violating couplings from nucleon decay amplitudes. Although trilinear R-parity violating couplings do not induce too rapid a proton decay, squark-exchange diagrams combining bilinear and dimension-5 R-parity violating operators still induce nucleon decay. We will obtain a big picture of allowed region of parameter space of bilinear–dimension-5 R-parity violation, and find that natural expectation of these parameters that follows from the framework in section 2 fits well within the allowed region.

If time is limited, it is best to read only summary sections 2.6 and 3.5, skipping all the rest. It is also possible to read sections 2 and 3 separately, as the materials in these sections do not require perfect understanding of the contents of the other.

2 Theoretical Framework

Section 2.1 provides basic knowledge in Calabi–Yau compactification of the Heterotic $E_8 \times E'_8$ string theory that is necessary later in section 2. We include this mini-review just to make this paper self-contained. Thus, there should be no problem in skipping section 2.1 and proceeding directly to 2.2.

2.1 Mini-review on Heterotic String Compactification

Origin of Gauge Fields and Matter Multiplets

Effective theories on 3+1 dimensions can be obtained by compactifying the Heterotic $E_8 \times E'_8$ string theory on a six-dimensional manifold X . $\mathcal{N} = 1$ supersymmetry is preserved in a low-energy effective theory when X is a Calabi–Yau 3-fold (and hence is a complex manifold). Local complex coordinates are denoted by z^α ($\alpha = 1, 2, 3$).

The gauge group $E_8 \times E'_8$ can be reduced virtually to any subgroups in low-energy effective theories by turning on non-trivial gauge-field background on X . We will refer to the gauge field background³ as V . The gauge group of the effective theory is $SU(5)_{\text{GUT}}$ for unified theories, if the background gauge field configuration is contained within $SU(5)' \times E'_8 \subset E_8 \times E'_8$, where $SU(5)'$ commutes with $SU(5)_{\text{GUT}}$ within E_8 .

A super Yang–Mills multiplet on 9+1 dimensions consists of a vector field A_M and a gauge fermion Ψ . A vector field on 9+1 dimensions, $A_M(x, y)$ ($M = 0, \dots, 9$), is decomposed into $A_\mu(x, y)$ ($\mu = 0, \dots, 3$) and $A_{\bar{\alpha}}(x, y)$ ($\alpha = 1, 2, 3$); $A_\alpha(x, y)$ is a complex conjugate of $A_{\bar{\alpha}}$ and is not an independent degree of freedom. Here, x^μ ($\mu = 0, 1, 2, 3$) denote four Minkowski coordinates, and y^m ($m = 1, \dots, 6$) (or simply y) six real local coordinates on X , equivalent of three complex coordinates $(z^\alpha, \bar{z}^{\bar{\alpha}})$ ($\alpha = 1, 2, 3$).

Those fields on 9+1 dimensions are decomposed into infinite degrees of freedom on 3+1 dimensions: their Kaluza–Klein decompositions are given by

$$A_\mu(x, y) = \sum_I A_{I;\mu}(x) \varphi_I(y), \quad (2.1)$$

$$A_{\bar{\alpha}}(x, y) = g_{\text{YM}} \sum_J \phi_J(x) \varphi_{J;\bar{\alpha}}(y), \quad (2.2)$$

using mode functions $\varphi_I(y)$ and $\varphi_{J;\bar{\alpha}}(y)$: $A_{I;\mu}(x)$ and $\phi_J(x)$ are vector and complex scalar fields on 3+1 dimensions, respectively. We factored out g_{YM} in (2.2) in setting the normalization of

³To be more precise, V means a vector bundle in the fundamental representation that a gauge field background in an $SU(n)$ subgroup of E_8 defines.

the mode functions $\varphi_{J;\bar{\alpha}}$; g_{YM} is the gauge coupling constant of the $\text{SU}(5)_{\text{GUT}}$ effective theory on 3+1 dimensions. A gauge fermion on 9+1 dimensions,

$$\Psi(x, y) = (\Psi_\alpha^a(x, y), \Psi_{\dot{a}}^{\dot{\alpha}}(x, y)), \quad (2.3)$$

splits into $a = 1, 2, 3$ part and $a = 4$ part on a Calabi–Yau 3-fold X , where α and $\dot{\alpha}$ denote doublet indices of left- and right-handed spinors of $\text{SO}(3, 1)$, and a label four different entries of spinor representations $\mathbf{4} + \bar{\mathbf{4}}$ of local Lorentz symmetry $\text{SO}(6)$. The two parts have Kaluza–Klein mode decompositions separately:

$$(\Psi_\alpha^{a=4}, \Psi_{\dot{a}=4}^{\dot{\alpha}})(x, y) = \sum_I (\lambda_{I;\alpha}(x) \chi_I(y), \bar{\lambda}^{\dot{\alpha}}(x) \tilde{\chi}_I(y)), \quad (2.4)$$

$$(\Psi_\alpha^a, \Psi_{\dot{a}}^{\dot{\alpha}})(x, y) = g_{\text{YM}} \sum_J (\psi_{J;\alpha}(x) \chi_J^a(y), \bar{\psi}_J^{\dot{\alpha}}(x) \chi_{J;\dot{a}}(y)) \quad (a = 1, 2, 3). \quad (2.5)$$

Mode functions $e_{\bar{\alpha}a} \chi_J^a(y)$ (being multiplied by a sechsbein⁴ $e_{\bar{\alpha}a}$) are proportional to those of the vector fields $\varphi_{J;\bar{\alpha}}(y)$ in a supersymmetric compactification, and $A_{\bar{\alpha}}(x, y)$ and $\Psi_\alpha^a(x, y)$ are grouped into a Kaluza–Klein tower of chiral multiplets $\Phi_I(x, \theta, \bar{\theta})$:

$$A_{\bar{\alpha}}(x, z, \bar{z}, \theta, \bar{\theta}) d\bar{z}^{\bar{\alpha}} \equiv \sum_J (\phi_J + \theta \psi_J + \cdots)(x) \varphi_{J;\bar{\alpha}}(y) d\bar{z}^{\bar{\alpha}} \equiv \sum_J \Phi_J \varphi_{J;\bar{\alpha}} d\bar{z}^{\bar{\alpha}}. \quad (2.6)$$

The remaining A_μ ($\mu = 0, \dots, 3$) part and $\Psi_\alpha^{a=4}$ part are also grouped into a tower of vector multiplets $V_I(x, \theta, \bar{\theta})$:

$$V(x, z, \bar{z}, \theta, \bar{\theta}) \equiv \sum_I (\theta \sigma^\mu \bar{\theta} A_{I;\mu} + \bar{\theta}^2 \theta \lambda_I + \cdots)(x) \varphi_I(y) \equiv \sum_I V_I \varphi_I. \quad (2.7)$$

As we assume that gauge-field background is non-vanishing, mode functions $\varphi_{J;\bar{\alpha}}(y)$ and $\varphi_I(y)$ are not the same everywhere in E_8 . The vector field and gauge fermion of the E_8 super Yang–Mills theory are in the adjoint representation of E_8 . They split into irreducible components

$$\mathbf{248} \rightarrow (\mathbf{adj.}, \mathbf{1}) + (\mathbf{1}, \mathbf{adj.}) + [(\mathbf{5}, \mathbf{10}) + (\mathbf{10}, \bar{\mathbf{5}})] + \text{h.c.} \quad (2.8)$$

of $\text{SU}(5)' \times \text{SU}(5)_{\text{GUT}} \subset E_8$. Each irreducible component has its own Kaluza–Klein decomposition (2.6, 2.7): Mode functions in the (R', R) -irreducible component are determined by mode equations on the gauge-field background in the R' representation, and hence the spectrum and

⁴Hermitian metric $h_{\alpha\bar{\alpha}}$ of a Kähler manifold X is given by $h_{\alpha\bar{\alpha}} = \sum_{a=1}^3 e_\alpha^a e_{\bar{\alpha}a}$.

decomposition of one irreducible component are different from those of another. Therefore we use such notations as $(R', R)_J$ or R_J , instead of Φ_J for chiral multiplets.

On the $SU(5)'$ gauge-field background, massless vector multiplets are found only in the $(\mathbf{1}, \mathbf{adj.})$ component. Massless chiral multiplets in the $SU(5)_{\text{GUT}}-R$ representation are in one to one correspondence with the zero modes $\varphi_{J;\bar{\alpha}}(y)$ in the (R', R) irreducible component. Difference between the number of massless chiral multiplets—net chirality—in a Hermitian conjugate pair of irreducible components, $(R', R)-(\bar{R}', \bar{R})$, is determined only by topology of the background gauge-field configuration V . Phenomenological request is to find a topology of (X, V) so that the net chirality in the $(\mathbf{5}, \mathbf{10})-(\bar{\mathbf{5}}, \bar{\mathbf{10}})$ sector is three. It then follows that the net chirality in the $(\mathbf{10}, \bar{\mathbf{5}})-(\bar{\mathbf{10}}, \mathbf{5})$ sector also becomes three; low-energy effective theories cannot be anomalous if they are obtained by compactifying an anomaly free theory in higher dimensions.

Vector-like Massless Pair

Topology of (X, V) determines the net chirality, but the number of massless chiral multiplets of each irreducible component can vary for a continuous deformation of gauge field background: There can be $3+m$ massless chiral multiplets in the $\bar{\mathbf{5}}$ representation and m in $\mathbf{5}$; m can change while keeping the net chirality $(3+m)-m=3$. For a given topology of (X, V) , however, there is still a minimum number of m for the $\bar{\mathbf{5}}-\mathbf{5}$ sector, and it is not necessarily zero.⁵ See [11] for an explicit model, where $m \geq 34$ for the $\bar{\mathbf{5}}-\mathbf{5}$ sector, while there can be no extra massless vector-like pair in the $\mathbf{10}-\bar{\mathbf{10}}$ sector. Similar examples can be found in the literature studying spectra on orbifold compactifications. So far, top down principles have been unable to determine the topology of (X, V) , and hence the minimum number of m 's.

There are a few bottom-up constraints on the number of vector-like massless pairs. First of all, there should be at least one vector-like pair in the doublet part of the $\bar{\mathbf{5}}-\mathbf{5}$ sector.⁶ They are identified with the two Higgs doublets of the minimal supersymmetric standard model (MSSM). If there were too many $SU(5)_{\text{GUT}}$ -charged vector-like pairs, however, they would contribute to beta functions of the MSSM gauge coupling constants. Perturbative gauge coupling unification, one of the most important motivations of low-energy supersymmetry, would not be maintained any more. As long as the “massless” pairs have masses of the order of SUSY-breaking scale or higher,⁷ they have not showed up in experiments, and moderate number of them are tolerable

⁵The same thing can happen for the multiplets in the $SU(5)_{\text{GUT}}-\mathbf{10}$ and $\bar{\mathbf{10}}$ representations, in principle.

⁶ $SU(5)_{\text{GUT}}$ symmetry can be broken either by a Wilson line [12] or by a line bundle [13]. Spectra and mode functions in a given $SU(5)_{\text{GUT}}$ representation can be different for different irreducible components of the MSSM gauge group. Hence m can be 1 for the doublet part, while $m=0$ for triplets. We maintain $SU(5)_{\text{GUT}}$ notations in many places in this paper, mainly to avoid cluttered equations, at the cost of sacrificing rigorousness and unambiguity.

⁷ Since all the argument based on Calabi–Yau compactification preserves $\mathcal{N}=1$ supersymmetry, a vector-like

in phenomenology.⁸

Superpotential of the Heterotic theory is given by [14]

$$W = cM_G^3 \int_X \Omega \wedge \text{tr}_{\text{adj.}} \left(AdA - \frac{2}{3}iAAA \right), \quad (2.9)$$

which is valid for all the Kaluza–Klein modes Φ_J in (2.6) [15]. Here, c is a numerical constant of order unity,⁹ $M_G \simeq 2.4 \times 10^{18}$ GeV and Ω is a dimensionless holomorphic 3-form of a Calabi–Yau 3-fold X . We treat Ω purely as a background, as we will only discuss what happens within the visible sector E_8 in this article.

Kaluza–Klein masses of infinite chiral multiplets originate from $d - 2i \langle A \rangle$, second derivative of (2.9) with respect to A . Writing the superpotential (2.9) fully in terms of $D = 4$ chiral multiplets, we have bilinear (Kaluza–Klein mass) and trilinear terms:

$$W \supset M_5(\mathbf{10}, \bar{\mathbf{5}})_*(\overline{\mathbf{10}}, \mathbf{5})_* + M_{10}(\mathbf{5}, \mathbf{10})_*(\bar{\mathbf{5}}, \overline{\mathbf{10}})_* + M_1(\mathbf{adj.}, \mathbf{1})_*(\mathbf{adj.}, \mathbf{1})_*, \quad (2.11)$$

$$+ y^u(\mathbf{5}, \mathbf{10})(\mathbf{5}, \mathbf{10})(\overline{\mathbf{10}}, \mathbf{5}) + y^d(\mathbf{10}, \bar{\mathbf{5}})(\mathbf{5}, \mathbf{10})(\mathbf{10}, \bar{\mathbf{5}}), \quad (2.12)$$

$$+ y^\nu(\mathbf{10}, \bar{\mathbf{5}})(\mathbf{adj.}, \mathbf{1})(\overline{\mathbf{10}}, \mathbf{5}) + y'^\nu(\bar{\mathbf{5}}, \overline{\mathbf{10}})(\mathbf{adj.}, \mathbf{1})(\mathbf{5}, \mathbf{10}), \quad (2.13)$$

$$+ y''^\nu(\mathbf{adj.}, \mathbf{1})(\mathbf{adj.}, \mathbf{1})(\mathbf{adj.}, \mathbf{1}). \quad (2.14)$$

Here, multiplets $(R', R)_*$ represent infinitely many massive chiral multiplets $(R', R)_I$ in the R representation of $\text{SU}(5)_{\text{GUT}}$; we will use $(R', R)_0$ when we refer specifically to massless modes. Those without any $*$ or $_0$ stand for both. Chiral multiplets $(\mathbf{adj.}, \mathbf{1})$ are $\text{SU}(5)_{\text{GUT}}$ singlets, and $(\mathbf{adj.}, \mathbf{1})_0$ correspond to gauge-field moduli.¹⁰ Mass matrices such as M_5 and M_{10} are of infinite rank. Rank of M_5 may be reduced at certain subset of gauge field moduli space,

pair of multiplets that are massless at a supersymmetric limit may have masses when the supersymmetry is weakly broken.

⁸Constraints such as FCNC and nucleon decay depend on couplings that those extra particles have with the chiral multiplets in the MSSM. Although such constraints can be very severe, we do not discuss them in this article.

⁹Combined with a Kähler potential,

$$\frac{K}{M_G^2} = -\ln(S + S^\dagger) - \ln\left(\int_X (T + T^\dagger)^3\right) - \ln\left(\int_X \Omega \wedge \overline{\Omega}\right), \quad (2.10)$$

this superpotential reproduces a part of the gaugino kinetic term (among other things) with the right dependence on α' and g_s .

¹⁰ Separation between the mass terms and the last two lines is not well-defined; this is because continuous deformation of a gauge field background along its moduli space corresponds to changing vev's of massless chiral multiplets $(\mathbf{adj.}, \mathbf{1})_0$, and extra mass terms arise from the last two lines.

where extra pairs of chiral multiplets in the $SU(5)_{\text{GUT}}\text{-}\bar{\mathbf{5}} + \mathbf{5}$ representations (or possibly in the $\mathbf{10} + \overline{\mathbf{10}}$) are in the low-energy spectrum [11].

Yukawa Interactions

The last three lines, (2.12–2.14), are trilinear interactions involving massless and/or massive chiral multiplets, with coupling constants given by overlap integration of relevant mode functions. Trilinear interactions involving only massless modes are directly relevant to low-energy physics. The first term of (2.12) contains up-type Yukawa couplings $W \ni y^u \mathbf{10} \mathbf{10} H(\mathbf{5})$, and the second term down-type/charged-lepton Yukawa couplings $W \ni y^d \bar{\mathbf{5}} \mathbf{10} \bar{H}(\bar{\mathbf{5}})$. Neutrino Yukawa couplings $W \ni y^\nu \bar{\mathbf{5}} \bar{N} H(\mathbf{5})$ can only be found in the first term of (2.13), and hence the chiral multiplets for right-handed neutrinos are identified with (a subset of) gauge-field moduli,¹¹ $(\mathbf{adj.}, \mathbf{1})_0$ [16].

As we have introduced no distinction between $H_d \subset \bar{H}(\bar{\mathbf{5}})$ and $L \subset \bar{\mathbf{5}}$'s, however, the second term in (2.12) generically contains the trilinear R-parity violating operators (1.1) as well. Thus, a generic (X, V) is not acceptable phenomenologically because of the dimension-4 proton decay problem.

R parity

The most popular solution to this problem is to impose an R parity. In terms of compactification of a super Yang–Mills theory on 9+1 dimensions, this is to assume a \mathbb{Z}_2 symmetry in (X, V) . We assign odd R-parity for three chiral multiplets $\mathbf{10}_i = (Q_i, \bar{U}_i, \bar{E}_i)$ ($i = 1, 2, 3$) in bottom-up model building, but it corresponds to assuming that there are three massless states in the $(\mathbf{5}, \mathbf{10})^-$ irreducible component in this context. One further need to assume that there are none in the other irreducible components $(\mathbf{5}, \mathbf{10})^+$, $(\bar{\mathbf{5}}, \overline{\mathbf{10}})^+$ and $(\bar{\mathbf{5}}, \overline{\mathbf{10}})^-$. Similar assumptions have to be made for irreducible components that are in the $\mathbf{5}$ and $\bar{\mathbf{5}}$ representations of $SU(5)_{\text{GUT}}$. See Table 1 for more details.

Both moduli $(\mathbf{adj.}, \mathbf{1})_0^+$ and $(\mathbf{adj.}, \mathbf{1})_0^-$ may have non-vanishing vev's without breaking $SU(5)_{\text{GUT}}$ symmetry, but non-vanishing vev's of the latter break the \mathbb{Z}_2 symmetry. Thus, none of $(\mathbf{adj.}, \mathbf{1})_0^-$'s should develop a vev in order to maintain an unbroken R-parity.

With such an unbroken \mathbb{Z}_2 symmetry of (X, V) , down-type/charged-lepton Yukawa couplings

$$W \ni y^d (\mathbf{10}, \bar{\mathbf{5}})_0^- (\mathbf{5}, \mathbf{10})_0^- (\mathbf{10}, \bar{\mathbf{5}})_0^+ \quad (2.15)$$

do not vanish, yet the \mathbb{Z}_2 -odd operators

$$W \ni \lambda (\mathbf{10}, \bar{\mathbf{5}})_0^- (\mathbf{5}, \mathbf{10})_0^- (\mathbf{10}, \bar{\mathbf{5}})_0^- \quad (2.16)$$

¹¹Strictly speaking, right-handed neutrinos do not have to be identified with zero modes. See also the 3+2 model to be discussed in section 2.2.

chiral mult.	repr.	# of zero modes	zero modes in low energy
$\mathbf{10}_0, \mathbf{10}_*$	$(\bar{\mathbf{5}}, \wedge^2 \mathbf{5})^-$	3	$Q_i, \bar{U}_i, \bar{E}_i \ (i = 1, 2, 3)$
$\mathbf{10}_*^c$	$(\bar{\mathbf{5}}, \wedge^2 \mathbf{5})^-$	0	-
$\mathbf{10}'_*$	$(\bar{\mathbf{5}}, \wedge^2 \mathbf{5})^+$	0	-
$\mathbf{10}'_*^c$	$(\bar{\mathbf{5}}, \wedge^2 \mathbf{5})^+$	0	-
$\bar{H}(\bar{\mathbf{5}})_0, \bar{H}(\bar{\mathbf{5}})_*$	$(\wedge^2 \mathbf{5}, \bar{\mathbf{5}})^+$	$m = 0(\bar{\mathbf{3}}), 1(\mathbf{2})$	H_d
$H(\mathbf{5})_0, H(\mathbf{5})_*$	$(\wedge^2 \bar{\mathbf{5}}, \mathbf{5})^+$	$m = 0(\mathbf{3}), 1(\bar{\mathbf{2}})$	H_u
$\bar{\mathbf{5}}_0, \bar{\mathbf{5}}_*$	$(\wedge^2 \mathbf{5}, \bar{\mathbf{5}})^-$	3	$\bar{D}_i, L_i \ (i = 1, 2, 3)$
$\bar{\mathbf{5}}_*^c$	$(\wedge^2 \bar{\mathbf{5}}, \mathbf{5})^-$	0	-
\bar{N}_0, \bar{N}_*	$(\mathbf{adj.}, \mathbf{1})^-$	some	RH neutrinos
	$(\mathbf{adj.}, \mathbf{1})^+$	0	-

Table 1: List of chiral multiplets in scenarios with an R parity. In the third column, $m = 0$ for the $SU(3)_C$ -triplet parts and $m = 1$ for the $SU(2)_L$ -doublet parts in the third row of this table. We listed just the minimum number of chiral multiplets required in effective theory on 3+1 dimensions, ignoring a vector-like (almost) massless pair that can exist without phenomenological problems.

are absent because the overlap integrations for the couplings vanish.

2.2 4+1 Model and 3+2 Model

Gauge field configuration V in $SU(5)' \subset E_8$ on a Calabi–Yau 3-fold X should not be generic, since generic configuration gives rise to the R-parity violating trilinear operators (1.1), leading to too rapid proton decay. An R-parity preserving configuration (X, V) with a \mathbb{Z}_2 symmetry is an example of non-generic cases.

Discrete symmetries other than R parity have also been discussed in the literature as solutions to the proton decay problem. Phenomenological consequences of these theory can be different from those of R-parity preserving ones; the LSP may not be stable, for example. Despite the difference in phenomenology, all the solutions based on discrete symmetries—whether preserving R parity or not—are quite similar in philosophy. Enhanced discrete (or continuous) symmetries are left unbroken only at special points (or subsets) of moduli space (X, V) . Therefore, solutions based on discrete symmetries are based on a belief that some dynamics that we do not know today will eventually select out vacua with enhanced symmetries, and lift all the other part of moduli space. Certainly this belief is not without reason. CP symmetry does not have to be preserved in QCD, but once we knew that QCD instanton effects generate a potential

of axion, we understood that CP-preserving $\theta_{\text{eff.}} = 0$ is the minimum of axion potential. So, who can say that history does not repeat itself?

In this article, however, we neither resort to this belief, nor assume an unknown dynamics for vacuum selection. We will present an alternative solution to the dimension-4 proton decay problem, which holds at generic points in the gauge-field moduli space. This subsection describes a class of compactification that solves the problem. Key ideas were already written in [7, 8], but a few important clarifications are newly added in this article. We then move on in the rest of section 2 to discuss what kind of operators are to be expected in low-energy effective theories on 3+1 dimensions.

4+1 Model

Let us first suppose that a gauge field background is restricted to an $\text{SU}(4) \times \text{U}(1)_\chi$ subgroup of $\subset \text{SU}(5)' \subset E_8$. Then, the gauge symmetry is $(\text{SU}(5)_{\text{GUT}} \times \text{U}(1)_\chi)/\mathbb{Z}_5$ in an effective theory. $\text{U}(1)_\chi/\mathbb{Z}_5$ contains matter parity as a subgroup, $\mathbb{Z}_{10}/\mathbb{Z}_5 \simeq \mathbb{Z}_2$. Although the matter parity is not broken at this moment, we will see later that the restriction on the gauge field background is relaxed and the matter parity is broken, yet the trilinear matter parity violating operators (1.1) are absent.

Each irreducible component of the E_8 -**adj.** representation in (2.8) is further split up as the group of gauge field background (called structure group) is reduced from $\text{SU}(5)'$ to $\text{SU}(4) \times \text{U}(1)_\chi$:

$$(\mathbf{5}, \mathbf{10}) \rightarrow (\mathbf{4}, \mathbf{10})^{-1} + (\mathbf{1}, \mathbf{10})^{+4}, \quad (2.17)$$

$$(\wedge^2 \mathbf{5}, \bar{\mathbf{5}}) \rightarrow (\wedge^2 \mathbf{4}, \bar{\mathbf{5}})^{-2} + (\mathbf{4}, \bar{\mathbf{5}})^{+3}, \quad (2.18)$$

$$(\mathbf{adj.}, \mathbf{1}) \rightarrow (\mathbf{adj.}, \mathbf{1})^0 + (\mathbf{4}, \mathbf{1})^{-5} + (\bar{\mathbf{4}}, \mathbf{1})^{+5} + (\mathbf{1}, \mathbf{1})^0. \quad (2.19)$$

Massless modes are identified with various chiral multiplets of the MSSM as in Table 2 in a compactification with this class of gauge field background.¹² $H_u \subset H(\mathbf{5})$ and $H_d \subset \bar{H}(\bar{\mathbf{5}})$ are completely vector like, as in the scenario with an R parity. Massive modes—those that acquire masses through $W \ni \text{tr}(AdA - 2iA \langle A \rangle A)$ —have subscripts $*$, and massless modes have $_0$, like in Table 1. Topology of geometry and gauge field configuration on it, (X, V) , should be chosen so that the right number of massless modes are obtained.

¹²Here is our naming rule of various chiral multiplets. Since anti-chiral multiplets containing right-handed quarks and leptons are denoted by $\bar{U}^\dagger, \bar{D}^\dagger$ and \bar{E}^\dagger , we save \bar{N}^\dagger for right-handed neutrinos. Chiral multiplets Ψ and Ψ^c ($\Psi = \mathbf{10}, \mathbf{10}', \bar{\mathbf{5}}, \bar{\mathbf{N}}$, for example) arise from a Hermitian conjugate pair of irreducible components (R', R) and (\bar{R}', \bar{R}) in E_8 . The same rule is also applied to the 3+2 model. In the 4+1 model, therefore, $\bar{H}(\bar{\mathbf{5}}) = H(\mathbf{5})^c$, but this is not the case in the 3+2 model. Since gauge-field moduli $(\mathbf{adj.}, \mathbf{1})^0$ is a vector-like representation of $\text{SU}(4) \times \text{SU}(5)_{\text{GUT}} \times \text{U}(1)_\chi$, there is no distinction between Φ and Φ^c .

fields	representations	number of zero-modes	zero-modes at low energy
$\mathbf{10}_0, \mathbf{10}_*$	$(\mathbf{4}, \wedge^2 \mathbf{5})^{-1}$	3	$Q_i, \bar{U}_i, \bar{E}_i$ ($i = 1, 2, 3$)
$\mathbf{10}_*^c$	$(\bar{\mathbf{4}}, \wedge^2 \mathbf{5})^{+1}$	0	—
$\mathbf{10}'_*$	$(\mathbf{1}, \wedge^2 \mathbf{5})^{+4}$	0	—
$\mathbf{10}'_*^c$	$(\mathbf{1}, \wedge^2 \bar{\mathbf{5}})^{-4}$	0	—
$\bar{H}_0(\bar{\mathbf{5}}), \bar{H}_*(\bar{\mathbf{5}})$	$(\wedge^2 \mathbf{4}, \bar{\mathbf{5}})^{-2}$	$1(\mathbf{2}) / 0(\bar{\mathbf{3}})$	H_d
$H_0(\mathbf{5}), H_*(\mathbf{5})$	$(\wedge^2 \bar{\mathbf{4}}, \mathbf{5})^{+2}$	$1(\mathbf{2}) / 0(\mathbf{3})$	H_u
$\bar{\mathbf{5}}_0, \bar{\mathbf{5}}_*$	$(\mathbf{4}, \bar{\mathbf{5}})^{+3}$	3	\bar{D}_i, L_i ($i = 1, 2, 3$)
$\bar{\mathbf{5}}_*^c$	$(\bar{\mathbf{4}}, \mathbf{5})^{-3}$	0	—
\bar{N}_0, \bar{N}_*	$(\mathbf{4}, \mathbf{1})^{-5}$	some	heavy RH neutrinos
\bar{N}_0^c, \bar{N}_*^c	$(\bar{\mathbf{4}}, \mathbf{1})^{+5}$	$1 \leq$	absorbed by $U(1)_\chi$
Φ_0, Φ_*	$(\mathbf{adj.}, \mathbf{1})^0$	—	—

Table 2: List of chiral multiplets in the 4+1 model. Those with a subscript $_0$ are zero modes of Kaluza–Klein decomposition, and those with $_*$ infinitely many massive modes. Chiral multiplets without either one of the subscripts $_0$ or $_*$ that appear in the text represent both of them. The second column shows how the chiral multiplets transform under $SU(5)_{\text{GUT}} \supset SU(3)_C \times SU(2)_L \times U(1)_Y$ as well as underlying broken symmetries $SU(4) \times U(1)_\chi$ in this model. Only the minimum number of chiral multiplets are shown in the third column. The slashes in the third column represent the doublet-triplet splitting. For more, see the caption of Table 1 and footnote 18.

Superpotential is obtained by rewriting (2.9) in terms of chiral multiplets in the irreducible components:¹³

$$W = y_{(1)}^u \mathbf{10} \mathbf{10} H(\mathbf{5}) + y_{(2)}^u \mathbf{10} \mathbf{10}'_* \bar{\mathbf{5}}_*^c \quad (2.20)$$

$$+ y_{(3)}^u \mathbf{10}_*^c \mathbf{10}_*^c \bar{H} + y_{(4)}^u \mathbf{10}_*^c \mathbf{10}'_*^c \bar{\mathbf{5}} \quad (2.21)$$

$$+ y_{(1)}^d \bar{\mathbf{5}} \mathbf{10} \bar{H}(\bar{\mathbf{5}}) + y_{(2)}^d \bar{H}(\bar{\mathbf{5}}) \mathbf{10}'_* \bar{H}(\bar{\mathbf{5}}) \quad (2.22)$$

$$+ y_{(3)}^d \bar{\mathbf{5}}_*^c \mathbf{10}_*^c H(\mathbf{5}) + y_{(4)}^d H(\mathbf{5}) \mathbf{10}'_*^c H(\mathbf{5}) \quad (2.23)$$

$$+ y_{(1)}^{\nu} \bar{N} \bar{\mathbf{5}} H(\mathbf{5}) + y_{(2)}^{\nu} \bar{N}_*^c \bar{\mathbf{5}}_*^c \bar{H} + y_{(3)}^{\nu} \bar{H}(\bar{\mathbf{5}}) \Phi H(\mathbf{5}) + y_{(4)}^{\nu} \bar{\mathbf{5}} \Phi \bar{\mathbf{5}}_*^c \quad (2.24)$$

$$+ y_{(1)}^{\nu} \bar{N} \mathbf{10}'_* \mathbf{10}_*^c + y_{(2)}^{\nu} \bar{N}_*^c \mathbf{10}'_*^c \mathbf{10} + y_{(3)}^{\nu} \mathbf{10} \Phi \mathbf{10}_*^c + y_{(4)}^{\nu} \mathbf{10}'_* \Phi \mathbf{10}'_*^c \quad (2.25)$$

$$+ y_{(1)}^{\nu\nu} \bar{N} \Phi \bar{N}^c + y_{(2)}^{\nu\nu} \Phi \Phi \Phi \quad (2.26)$$

$$+ M_{\mathbf{10}} \mathbf{10}_* \mathbf{10}_*^c + M_{\mathbf{10}'} \mathbf{10}'_* \mathbf{10}'_*^c + M_H H_* \bar{H}_* + M_{\mathbf{5}} \bar{\mathbf{5}}_* \bar{\mathbf{5}}_*^c + M_N \bar{N}_* \bar{N}_*^c + M_\Phi \Phi_* \Phi_* \quad (2.27)$$

¹³ An equation (2.20–2.26) sets up notation for trilinear couplings in the 4+1 model. When we just simply use y^u and y^d in the text, however, they mean some of $y_{(1)-(4)}^u$ and $y_{(1)-(4)}^d$, respectively. Similarly, y^ν stands for some of $y_{(1)-(4)}^\nu$, $y_{(1)-(4)}^{\nu\nu}$ and $y_{(1)-(2)}^{\nu\nu}$.

Dropping all the interactions involving heavy multiplets, the Yukawa couplings of quarks and leptons remain in the superpotential above. On the other hand, trilinear R-parity violating operators (1.1) are not found anywhere in (2.20–2.26), because

$$(\mathbf{4}, \bar{\mathbf{5}})^{+3} \otimes (\mathbf{4}, \wedge^2 \mathbf{5})^{-1} \otimes (\mathbf{4}, \bar{\mathbf{5}})^{+3} \quad (2.28)$$

does not contain a singlet of $\text{SU}(4) \times \text{SU}(5)_{\text{GUT}} \times \text{U}(1)_\chi$.

The Fayet–Iliopoulos parameter of $\text{U}(1)_\chi$ symmetry is given by [13]

$$\xi_\chi = \frac{10M_G^2}{32\pi^2} \left[\frac{2\pi l_s^2}{\text{vol}(X)} \int_X c_1(L) \wedge J \wedge J - \frac{g_{\text{YM}}^2 e^{2\tilde{\phi}_4}}{2} \int_X c_1(L) \left(c_2(V) - \frac{1}{2} c_2(TX) \right) \right], \quad (2.29)$$

which depends on Kähler moduli J (volume moduli) in the tree-level contribution (the first term) and dilaton expectation value at the 1-loop level (the second term: a piece well-known since 1980’s). $l_s = 2\pi\sqrt{\alpha'}$ is the string length. See [13, 7] for details of the convention. As long as moduli fields J and $\tilde{\phi}_4$ are stabilized by potential other than the D-term of $\text{U}(1)_\chi$ symmetry, there is no reason to believe that the tree and 1-loop contributions cancel one another. Thus, ξ_χ is not expected to vanish. If it is negative, then $+5|\overline{N}^c|^2$ in the D-term potential may absorb ξ_χ to restore supersymmetry. This scenario is called 4+1 model.¹⁴ The matter parity $\mathbb{Z}_{10}/\mathbb{Z}_5 \simeq \mathbb{Z}_2$ is broken by an expectation value of \overline{N}^c .

An order-of-magnitude estimate of $\text{U}(1)_\chi$ breaking vev $\langle \overline{N}^c \rangle$ was obtained in [7]. We assume that there is no significant cancellation between the tree and 1-loop terms in (2.29), and that a Calabi–Yau manifold X is “isotropic”, that is, its volume moduli are characterized by only one typical radius R . Then we can roughly estimate the Fayet–Iliopoulos parameter ξ_χ in terms of Kaluza–Klein scale $M_{\text{KK}} \sim 1/R$. ξ_χ in turn determines a vev of \overline{N}^c . Canonically normalized \overline{N}^c typically develops a vev of order

$$|\langle \overline{N}^c \rangle|^2 \approx \frac{1}{4\alpha_{\text{GUT}}} \frac{1}{R^2}, \quad (2.31)$$

which is roughly around the Kaluza–Klein scale [7].

¹⁴ The 4+1 model corresponds to turning on a rank-5 vector bundle V that is given by an extension of a rank-4 vector bundle U_4 by a line bundle L :

$$0 \rightarrow L \rightarrow V \rightarrow U_4 \rightarrow 0. \quad (2.30)$$

L is a sub-bundle of V , and $L \otimes V$ and $\wedge^2 \overline{U}_4$ are sub-bundles of $\wedge^2 V$ and $\wedge^2 \overline{V}$, respectively.

Such a large vev of \overline{N}^c generates Majorana mass terms of right-handed neutrinos \overline{N} . Once massive chiral multiplets Φ_* 's are integrated out, an effective interaction

$$W \ni \frac{(y''_{(1)\nu})^2}{M_\Phi} \overline{N}^c \overline{N} \overline{N}^c \overline{N} \quad (2.32)$$

is generated [17]. It is also known [18] that world-sheet instanton effects generate

$$W \ni e^{-T} \overline{27} \overline{27} 27 27 \quad (2.33)$$

in some compactifications of the Heterotic string theory with an unbroken E_6 gauge group, and (2.33) contains an interaction of the form

$$W \ni \frac{1}{M_*} \overline{N}^c \overline{N}^c \overline{N} \overline{N}. \quad (2.34)$$

Once the interaction of this form is generated, either from (2.32) or (2.33), then non-vanishing vev of \overline{N}^c provides Majorana mass terms of right-handed neutrinos [19], with masses of order

$$M_R = \frac{\langle \overline{N}^c \rangle^2}{M_*}; \quad \frac{1}{M_*} \approx \max \left(\frac{(y''_{(1)\nu})^2}{M_\Phi}, e^{-T} \right). \quad (2.35)$$

Majorana right-handed neutrinos generate small masses of left-handed neutrinos through the see-saw mechanism. At the same time, a flat direction $5|\overline{N}^c|^2 - 5|\overline{N}|^2 + \xi_\chi = 0$ is lifted because of the Majorana mass terms. Without a fine-tuning of expectation values of gauge-field moduli, we can obtain $\langle \overline{N}^c \rangle \neq 0$ while $\langle \overline{N} \rangle = 0$; $\langle \overline{N}^c \rangle \neq 0$ is crucial for neutrino masses as we have seen above, and $\langle \overline{N} \rangle = 0$ is crucial for the absence of trilinear R-parity violation [7].

In the presence of non-vanishing $\langle \overline{N}^c \rangle$, trilinear terms of (2.24–2.26) involving \overline{N}^c gives rise to extra mass terms. We have already seen how the mass matrix in the $SU(5)_{\text{GUT}}$ -singlet sector is deformed; an interaction (2.32) with \overline{N}^c replaced by their vev's is regarded as a mass term obtained after diagonalizing the mass matrix of Φ and \overline{N} [17]. Let us now look at how $\langle \overline{N}^c \rangle \neq 0$ modifies mass matrices of $SU(5)_{\text{GUT}}$ -charged multiplets.

In principle, mass eigenstates in a given representation of $SU(5)_{\text{GUT}}$ are mixture of states with different $U(1)_\chi$ charges, because the $U(1)_\chi$ symmetry is already broken spontaneously. In the $SU(5)_{\text{GUT}}-(\mathbf{10} + \overline{\mathbf{10}})$ sector, the mass matrix becomes

$$\begin{pmatrix} \mathbf{10}_*^c & \mathbf{10}'_*{}^c \end{pmatrix} \left(\frac{M_{\mathbf{10}}}{y'_{(2)\nu} \langle \overline{N}^c \rangle} \middle| \frac{M_{\mathbf{10}'}}{y'_{(2)\nu} \langle \overline{N}^c \rangle} \right) \begin{pmatrix} \mathbf{10}_0 \\ \mathbf{10}_* \\ \mathbf{10}'_* \end{pmatrix}. \quad (2.36)$$

By this 2×3 matrix, we actually mean an $(\infty + \infty) \times (3 + \infty + \infty)$ matrix for cases of practical interest. $U(1)_\chi$ eigenstates and mass eigenstates are related by a basis transformation¹⁵

$$\begin{pmatrix} \mathbf{10}_0 \\ \mathbf{10}_* \\ \mathbf{10}'_* \end{pmatrix} = \begin{pmatrix} \frac{M_{\mathbf{10}'}}{\sqrt{|M_{\mathbf{10}'}|^2 + |y'_{(2)} \langle \overline{N}^c \rangle|^2}} & * & * \\ 0 & * & * \\ -\frac{y'_{(2)} \langle \overline{N}^c \rangle}{\sqrt{|M_{\mathbf{10}'}|^2 + |y'_{(2)} \langle \overline{N}^c \rangle|^2}} & * & * \end{pmatrix} \begin{pmatrix} \hat{\mathbf{10}}_0 \\ \hat{\mathbf{10}}_* \\ \hat{\mathbf{10}}'_* \end{pmatrix}, \quad (2.37)$$

where $\hat{\mathbf{10}}_*$ are massive mass eigenstates, which consist mainly of linear combination of $\mathbf{10}_*$ and $\mathbf{10}'_*$, and $\hat{\mathbf{10}}_0$ are massless degrees of freedom in the presence of non-vanishing vev of \overline{N}^c . Both massive and massless eigenstates $\hat{\mathbf{10}}_*$ and $\hat{\mathbf{10}}_0$ are mixtures of states $\mathbf{10}$ and $\mathbf{10}'$ with different $U(1)_\chi$ charges.

Mass matrix in the $SU(5)_{\text{GUT}}-(\bar{\mathbf{5}} + \mathbf{5})$ sector is given by

$$\begin{pmatrix} H_0 & H_* & \bar{\mathbf{5}}_*^c \end{pmatrix} \left(\begin{array}{c|cc} 0 & & \\ \hline M_H & & \\ y'_{(2)} \langle \overline{N}^c \rangle & y'_{(2)} \langle \overline{N}^c \rangle & 0 \end{array} \middle| \begin{array}{c} M_{\mathbf{5}} \end{array} \right) \begin{pmatrix} \bar{H}_0 \\ \bar{H}_* \\ \bar{\mathbf{5}}_0 \\ \bar{\mathbf{5}}_* \end{pmatrix}, \quad (2.38)$$

and one finds that the $U(1)_\chi$ eigenstates contain massless (and massive) eigenstates as in

$$\begin{pmatrix} H_0 \\ H_* \\ \bar{\mathbf{5}}_*^c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix} \begin{pmatrix} \hat{H}_0 \\ \hat{\mathbf{5}}_* \\ \hat{\mathbf{5}}_* \end{pmatrix}, \quad (2.39)$$

$$\begin{pmatrix} \bar{H}_0 \\ \bar{H}_* \\ \bar{\mathbf{5}}_0 \\ \bar{\mathbf{5}}_* \end{pmatrix} = \begin{pmatrix} \frac{M_{\mathbf{5}}}{\sqrt{|M_{\mathbf{5}}|^2 + |y'_{(2)} \langle \overline{N}^c \rangle|^2}} & 0 & * & * \\ 0 & 0 & * & * \\ 0 & 1 & 0 & 0 \\ -\frac{y'_{(2)} \langle \overline{N}^c \rangle}{\sqrt{|M_{\mathbf{10}'}|^2 + |y'_{(2)} \langle \overline{N}^c \rangle|^2}} & 0 & * & * \end{pmatrix} \begin{pmatrix} \hat{\bar{H}}_0 \\ \hat{\bar{\mathbf{5}}}_0 \\ \hat{\bar{\mathbf{5}}}_* \\ \hat{\bar{\mathbf{5}}}_* \end{pmatrix}, \quad (2.40)$$

where $\hat{\bar{\mathbf{5}}}_*$ and $\hat{\mathbf{5}}_*$ are massive mass eigenstates and \hat{H}_0 , $\hat{\bar{\mathbf{5}}}_0$ and $\hat{\bar{H}}_0$ massless degrees of freedom. All the massive mass-eigenstates are mixture of states with different $U(1)_\chi$ charges, and so is a massless eigenstate $\hat{\bar{H}}(\bar{\mathbf{5}})_0$. But, other massless eigenstates, $\hat{H}(\mathbf{5})_0$ and $\hat{\bar{\mathbf{5}}}_0$, remain pure $U(1)_\chi$ eigenstates, $H(\mathbf{5})_0$ and $\bar{\mathbf{5}}_0$, respectively.¹⁶

¹⁵Here, we are not very careful in defining the phase of mass eigenstates.

¹⁶ The disparity between the natures of these massless modes stems from existence of well-defined subbundles L , $L \otimes U_4$ and $\wedge^2 \overline{U}_4$ that we mentioned in footnote 14. Massless eigenstates that remain pure $U(1)_\chi$ -eigenstates, namely, $\hat{\bar{\mathbf{5}}}_0$ and \hat{H}_0 , are characterized as $H^1(X; L \otimes V) \subset H^1(X; \wedge^2 V)$ and $H^1(X; \wedge^2 \overline{U}_4) \subset H^1(X; \wedge^2 \overline{V})$. Other massless eigenstates such as $\hat{\mathbf{10}}_0$ or $\hat{\bar{H}}_0$ do not have such characterization associated with subbundles [7].

It is important to note that there is a strict rule on the mixing of massless eigenstates. Massless eigenstates have their own $U(1)_\chi$ charges in a $\langle \overline{N}^c \rangle \rightarrow 0$ limit. When $\langle \overline{N}^c \rangle$ does not vanish, they can pick up interactions of states with different $U(1)_\chi$ charges, only when holomorphic insertion of $\langle \overline{N}^c \rangle$ can supply the right $U(1)_\chi$ charge [7, 8]. An \bar{H} -like massless eigenstate $\hat{\bar{H}}_0$ have a non-vanishing $\bar{\mathbf{5}}$ component—(4, 1) entry of the mixing matrix (2.40)—because $\langle \overline{N}^c \rangle \bar{H}$ has the same $U(1)_\chi$ charge as $\bar{\mathbf{5}}$. On the other hand, $\bar{\mathbf{5}}$ -like massless eigenstates $\hat{\bar{\mathbf{5}}}_0$ do not have \bar{H} components—vanishing (2, 2) entry of (2.40)—because $\langle \overline{N}^c \rangle \bar{\mathbf{5}}$ does not have the same $U(1)_\chi$ charge as \bar{H} . Mixing of massless eigenstates $\hat{\mathbf{10}}_0$ is also understood this way.

Mixing matrices are important, because interactions of mass eigenstates are obtained by substituting (2.37) and (2.40) into (2.20–2.27). Since massive mass-eigenstates are generic mixtures of states with different $U(1)_\chi$ charges, the $U(1)_\chi$ symmetry is virtually powerless in controlling their interactions in the superpotential. Trilinear terms in the superpotential that involve only massless eigenstates, however, are still controlled by the $U(1)_\chi$ symmetry, because the mixing of massless eigenstates is under the rule above: $U(1)_\chi$ charge can be supplied only through holomorphic insertion of $\langle \overline{N}^c \rangle$. Three point interactions of massless states do not exist, if sums of $U(1)_\chi$ charges of these states in the $\langle \overline{N}^c \rangle \rightarrow 0$ limit are positive, because such interactions are not $U(1)_\chi$ invariant even after allowing holomorphic insertion of positively charged $\langle \overline{N}^c \rangle$. Here, we have a selection rule in the superpotential.

The R-parity violating trilinear interactions $\bar{\mathbf{5}}_0 \mathbf{10}_0 \bar{\mathbf{5}}_0$ have positive (+5) $U(1)_\chi$ charges, and holomorphic insertion of positively charged $\langle \overline{N}^c \rangle$ cannot make them neutral under $U(1)_\chi$. Thus, such interactions do not exist, and the 4+1 model is a solution to the dimension-4 proton decay problem. The $U(1)_\chi$ -charge counting allows an R-parity violating interaction of the form $W \ni \langle \overline{N}^c \rangle \bar{H}_0 \mathbf{10}_0 \bar{H}_0$. Indeed, by substituting (2.37) and (2.40) into (2.22), we find

$$W \ni y^d \frac{y^\nu \langle \overline{N}^c \rangle}{\sqrt{M^2 + |y^\nu \langle \overline{N}^c \rangle|^2}} \hat{\bar{H}}_0 \hat{\mathbf{10}}_0 \hat{\bar{H}}_0. \quad (2.41)$$

But this operator vanishes because of anti-symmetric contraction of $SU(5)_{\text{GUT}}$ indices and of the fact that there is only one down-type Higgs doublet in the MSSM. Therefore, trilinear R-parity violating operators are absent in the 4+1 model.

Finally, let us comment on what it takes in the 4+1 model to have two Higgs doublets in the low-energy spectrum, and a μ -term in the superpotential. In the doublet part¹⁷ of the

¹⁷ The doublet and triplet parts of $H(5)$ and $\bar{H}(\bar{5})$ have different KK towers, especially different numbers of massless modes, as discussed in footnote 6. Mass matrix (2.38) is intended to be for the doublet part. There are no massless modes for the triplet part, so the 1st row and the 1st and 3rd columns are dropped from the mass matrix.

$SU(5)_{\text{GUT}}\text{-}\mathbf{5} + \bar{\mathbf{5}}$ representations, we need three $\hat{\mathbf{5}}_0$ -like chiral multiplets L_i , one \hat{H}_0 -like H_d and one \hat{H}_0 -like H_u in the low-energy spectrum. This means that there should be at least 1 and $m' + 1$ zero modes in the doublet part of $H(\mathbf{5})$ and $\bar{H}(\bar{\mathbf{5}})$ sectors, and m' and 3 zero modes in the $\bar{\mathbf{5}}^c$ and $\bar{\mathbf{5}}$ sectors ($m' \geq 0$), respectively. $m' > 0$ is allowed when m' pairs of chiral multiplets $H_0\text{-}\bar{\mathbf{5}}_0^c$ acquire large masses¹⁸ through the second term in (2.24). Whether $m' = 0$ or not, we need a pair of vector-like massless chiral multiplets in the doublet part of $H(\mathbf{5})\text{-}\bar{H}(\bar{\mathbf{5}})$.

The existence of this extra massless vector-like pair is not guaranteed by topology. This pair, essentially the two Higgs doublets of the MSSM, can be in the low-energy spectrum for two possible reasons. The first possibility is that there is at least one extra pair of massless multiplets in the doublet part of $H(\mathbf{5})\text{-}\bar{H}(\bar{\mathbf{5}})$ sector for *generic* gauge-field configuration in a given topological class of (X, V) . See section 2.1 for more about this case. The second possibility is that the background gauge-field configuration of our world is somewhat *special*¹⁹ and an extra pair of multiplets becomes almost massless for the choice of background.

It is not hard for the second case for such two Higgs doublets to have a μ -term. Since they have trilinear couplings $W \ni \sum_I \Phi_{I,0} \hat{H}_0 \hat{\bar{H}}_0$ with gauge field moduli $\Phi_{I,0}$ (here the lower suffix I denotes the I -th modulus), μ -term is generated once the vev's of these moduli fields shift by of the order of SUSY-breaking scale. This is essentially the next-to-minimal SUSY Standard Model (NMSSM).²⁰ In the first case, such a trilinear coupling is absent (by definition). Instead, 1-loop diagrams generate such terms as

$$K \ni \frac{|y^u|^2}{16\pi^2} \frac{M_{10}^{*2}}{|M_{10}|^2} \hat{H}_0 \hat{\bar{H}}_0. \quad (2.42)$$

This one comes from the first diagram of Fig. 1, and there are also similar contributions from loops with multiplets in other representations; see Fig. 1. Remembering that holomorphic mass parameters in the superpotential have non-vanishing θ^2 components, at least by of order $M(1 + \theta^2 m_{3/2})$ when $\langle W \rangle^* / M_G^2 \neq 0$, μ -term and B_μ -term are generated at 1-loop. This is essentially the mechanism in [21], except that the vector-like massive multiplets in the loop are identified with Kaluza–Klein towers here. Note, however, that the μ - and B_μ -terms generated in this way are known to have a fine-tuning problem [22]. Thus, the problem may be an indication that the Higgs sector is a little more complicated than we imagine here, and that a bit of model building is necessary as in gauge- and anomaly-mediation scenarios.

¹⁸ Minimal choice $m' = 0$ is assumed implicitly in the mass matrix (2.38) and Table 2.

¹⁹ Possibly for an anthropic reason [20]. Special choice of gauge-field configuration could follow as a consequence of moduli stabilization, in principle, but no such claim based on an explicit string construction has been made so far.

²⁰Hence such moduli are free from moduli problem.

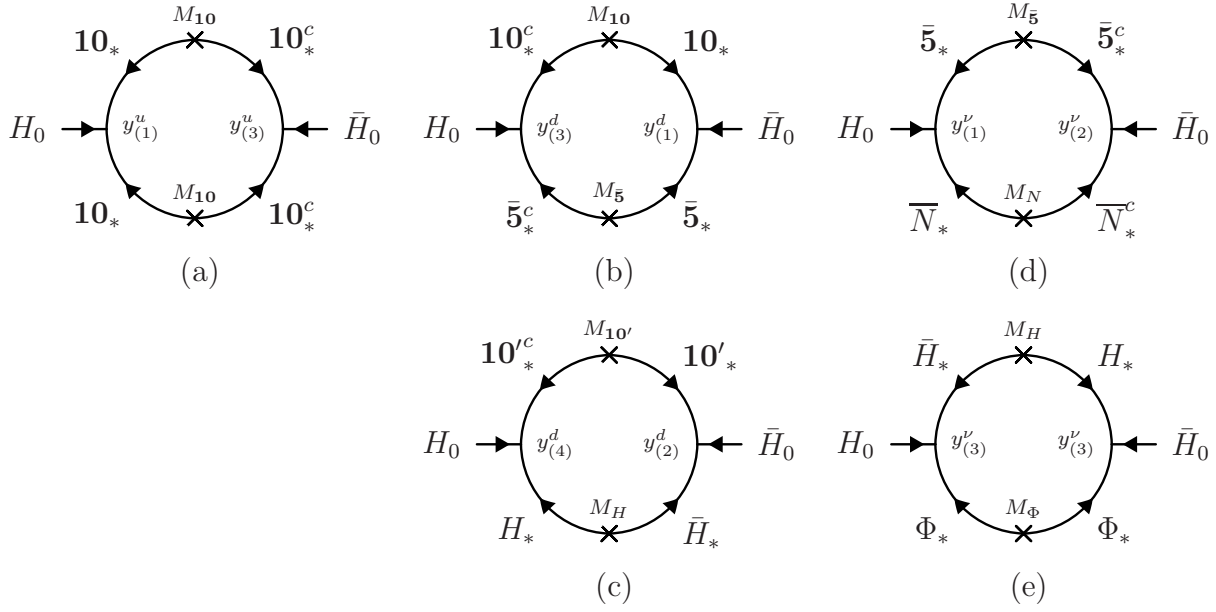


Figure 1: Super Feynman diagrams that generate μ -term in the 4+1 model. There are three different kinds of graphs; a pair of multiplets in the $\mathbf{10}$ and $\mathbf{10}$ representations of $SU(5)_{\text{GUT}}$ are running in the loop (a), those in the $\mathbf{\bar{10}}$ and $\mathbf{5}$ representations of the $SU(5)_{\text{GUT}}$ in the loop (b) (c), and finally, singlets and $\mathbf{5}$'s in the loop (d) (e).

fields	representations	number of zero-modes	zero-modes at low energy
$10_0, 10_*$	$(\mathbf{1}, \mathbf{2}, \wedge^2 \mathbf{5})^{-3}$	3	$Q_i, \bar{U}_i, \bar{E}_i$ ($i = 1, 2, 3$)
10_*^c	$(\mathbf{1}, \mathbf{2}, \wedge^2 \mathbf{5})^{+3}$	0	-
$10_*'$	$(\mathbf{3}, \mathbf{1}, \wedge^2 \mathbf{5})^{+2}$	0	-
$10_*'^c$	$(\bar{\mathbf{3}}, \mathbf{1}, \wedge^2 \mathbf{5})^{-2}$	0	-
$\bar{5}_0, \bar{5}_*$	$(\mathbf{3}, \mathbf{2}, \bar{\mathbf{5}})^{-1}$	$3(+b)$	\bar{D}_i, L_i ($i = 1, 2, 3$)
$\bar{5}_0^c, \bar{5}_*^c$	$(\mathbf{3}, \mathbf{2}, \bar{\mathbf{5}})^{+1}$	$0(+a)$	-
$H_0(\mathbf{5}), H_*(\mathbf{5})$	$(\mathbf{1}, \mathbf{1}, \mathbf{5})^{+6}$	$1(\mathbf{2}) / 0(\mathbf{3})$ ($+b$)	H_u
$H_*^c(\mathbf{5})$	$(\mathbf{1}, \mathbf{1}, \mathbf{5})^{-6}$	0	-
$\bar{H}_0(\bar{\mathbf{5}}), \bar{H}_*(\bar{\mathbf{5}})$	$(\wedge^2 \mathbf{3}, \mathbf{1}, \bar{\mathbf{5}})^{+4}$	$1(\mathbf{2}) / 0(\bar{\mathbf{3}})$ ($+a$)	H_d
$\bar{H}_*^c(\bar{\mathbf{5}})$	$(\wedge^2 \bar{\mathbf{3}}, \mathbf{1}, \bar{\mathbf{5}})^{-4}$	0	-
\bar{N}_0, \bar{N}_*	$(\mathbf{3}, \mathbf{2}, \mathbf{1})^{-5}$	1	heavy RH neutrino
\bar{N}_*^c	$(\mathbf{3}, \mathbf{2}, \mathbf{1})^{+5}$	0	-
Φ_0, Φ_*	$(\mathbf{adj.}, \mathbf{1}, \mathbf{1})^0$ $(\mathbf{1}, \mathbf{adj.}, \mathbf{1})^0$		

Table 3: List of fields for 3+2 model. Fields with subscripts $_0$ or $_*$ can be understood as before. The second column shows how the chiral multiplets transform under $SU(5)_{\text{GUT}}$ as well as underlying broken symmetries $SU(3) \times SU(2) \times U(1)_{\tilde{q}_7}$ in the 3+2 model. The slashes in the third column represent the doublet-triplet splitting. $a = b = 0$ in the minimal choice. We will find in section 2.5 that some effective operators can be enhanced depending on whether such a non-minimal pair ($a \neq 0$ or $b \neq 0$) of (eventually massive) chiral multiplets are in the spectrum or not.

3+2 Model

There are some variations of the 4+1 model [7], and one of them is called 3+2 model. Instead of restricting the structure group of gauge field background to $SU(4) \times U(1)_\chi \subset SU(5)'$ at the beginning, one can choose $SU(3) \times SU(2) \times U(1)_{\tilde{q}_7} \subset SU(5)'$ as the structure group. This restriction on the structure group is relaxed later by $U(1)_{\tilde{q}_7}$ -breaking vev, just like in the 4+1 model. Irreducible components in (2.8) split into

$$(\mathbf{5}, \wedge^2 \mathbf{5}) \rightarrow (\mathbf{1}, \mathbf{2}, \wedge^2 \mathbf{5})^{-3} + (\mathbf{3}, \mathbf{1}, \wedge^2 \mathbf{5})^{+2}, \quad (2.43)$$

$$(\wedge^2 \mathbf{5}, \bar{\mathbf{5}}) \rightarrow (\mathbf{1}, \wedge^2 \mathbf{2}, \bar{\mathbf{5}})^{-6} + (\mathbf{3}, \mathbf{2}, \bar{\mathbf{5}})^{-1} + (\wedge^2 \mathbf{3}, \mathbf{1}, \bar{\mathbf{5}})^{+4}, \quad (2.44)$$

$$(\mathbf{adj.}, \mathbf{1}) \rightarrow (\mathbf{adj.}, \mathbf{1}, \mathbf{1})^0 + (\mathbf{1}, \mathbf{adj.}, \mathbf{1})^0 + (\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})^{-5} + (\mathbf{3}, \bar{\mathbf{2}}, \mathbf{1})^{+5} \quad (2.45)$$

under $SU(3) \times SU(2) \times SU(5)_{\text{GUT}} \times U(1)_{\tilde{q}_7}$. Each irreducible component has its own towers of chiral and vector multiplets. Topology of (X, V) should be arranged so that the number of

zero modes of each component is appropriate for a low-energy effective theory; Table 3 shows the required number of zero modes as well as notation of chiral multiplets originating from each sector. Note that the tower containing the down-type Higgs doublet $\bar{H}(\bar{\mathbf{5}})$ is not the Hermitian conjugate of that containing the up-type Higgs doublet $H(\mathbf{5})$ in the 3+2 model, unlike in the 4+1 model.

Superpotential of the 3+2 model is given by rewriting (2.9):

$$W = y_{(1)}^u \mathbf{10} \mathbf{10} H + y_{(2)}^u \mathbf{10} \mathbf{10}'_* \bar{\mathbf{5}}_*^c + y_{(3)}^u \mathbf{10}'_* \mathbf{10}'_* \bar{H}_*^c \quad (2.46)$$

$$+ y_{(4)}^u \mathbf{10}_*^c \mathbf{10}_*^c H_*^c + y_{(5)}^u \mathbf{10}_*^c \mathbf{10}'_*^c \bar{\mathbf{5}} + y_{(6)}^u \mathbf{10}'_*^c \mathbf{10}'_*^c \bar{H} \quad (2.47)$$

$$+ y_{(1)}^d \bar{H} \mathbf{10} \bar{\mathbf{5}} + y_{(2)}^d \bar{H} \mathbf{10}'_* H_*^c + y_{(3)}^d \bar{\mathbf{5}} \mathbf{10}'_* \bar{\mathbf{5}} \quad (2.48)$$

$$+ y_{(4)}^d \bar{H}_*^c \mathbf{10}_*^c \bar{\mathbf{5}}_*^c + y_{(5)}^d \bar{H}_*^c \mathbf{10}'_*^c H + y_{(6)}^d \bar{\mathbf{5}}_*^c \mathbf{10}'_*^c \bar{\mathbf{5}}_*^c \quad (2.49)$$

$$+ y_{(1)}^\nu \bar{\mathbf{5}} \bar{N} H + y_{(2)}^\nu \bar{H} \bar{N} \bar{\mathbf{5}}^c + y_{(3)}^\nu \bar{\mathbf{5}}^c \bar{N}^c H_*^c + y_{(4)}^\nu \bar{H}_*^c \bar{N}^c \bar{\mathbf{5}} \quad (2.50)$$

$$+ y_{(5)}^\nu H \Phi H_*^c + y_{(6)}^\nu \bar{H} \Phi \bar{H}_*^c + y_{(7)}^\nu \bar{\mathbf{5}} \Phi \bar{\mathbf{5}}_*^c \quad (2.51)$$

$$+ y_{(1)}^{\nu'} \bar{N} \mathbf{10}_*^c \mathbf{10}'_* + y_{(2)}^{\nu'} \bar{N}^c \mathbf{10} \mathbf{10}'_*^c + y_{(3)}^{\nu'} \mathbf{10} \Phi \mathbf{10}_*^c + y_{(4)}^{\nu'} \mathbf{10}'_* \Phi \mathbf{10}'_*^c \quad (2.52)$$

$$+ y_{(1)}^{\nu''} \bar{N} \Phi \bar{N}^c + y_{(2)}^{\nu''} \Phi \Phi \Phi \quad (2.53)$$

$$+ M_{\mathbf{10}} \mathbf{10}_*^c \mathbf{10}_* + M_{\mathbf{10}'} \mathbf{10}'_*^c \mathbf{10}'_* + M_{\bar{H}} \bar{H}_*^c \bar{H}_* + M_{\bar{\mathbf{5}}} \bar{\mathbf{5}}_*^c \bar{\mathbf{5}}_* + M_H H_*^c H_* \quad (2.54)$$

$$+ M_N \bar{N}_*^c \bar{N}_* + M_\Phi \Phi_* \Phi_*. \quad (2.55)$$

The trilinear R-parity violating operators (1.1) are absent because

$$(\mathbf{3}, \mathbf{2}, \bar{\mathbf{5}})^{-1} \otimes (\mathbf{1}, \mathbf{2}, \wedge^2 \mathbf{5})^{-3} \otimes (\mathbf{3}, \mathbf{2}, \bar{\mathbf{5}})^{-1} \quad (2.56)$$

does not contain a singlet of $\text{SU}(3) \times \text{SU}(2) \times \text{SU}(5)_{\text{GUT}} \times \text{U}(1)_{\tilde{q}_7}$.

If the Fayet–Iliopoulos parameter of $\text{U}(1)_{\tilde{q}_7}$ is positive, only zero modes \bar{N}_0 , which carry negative $\text{U}(1)_{\tilde{q}_7}$ charge, develop non-vanishing expectation values. Majorana mass terms of \bar{N}^c are generated in the 3+2 model, just like those of \bar{N} are in the 4+1 model; see the discussion following (2.32, 2.33). Small masses of left-handed neutrinos are generated through the double see-saw mechanism, since vector-like mass terms of $\bar{N}_* \bar{N}_*^c$ are available from the last term of (2.55) [7]. Super-diagram description of the double see-saw mechanism is the one on the right-hand side of Fig. 2. The effective mass scale $M_{R,\text{eff.}}$ that characterizes the low-energy neutrino mass through $m_\nu \sim (y^\nu v)^2 / M_{R,\text{eff.}}$ is given by

$$M_{R,\text{eff.}} \sim \frac{M_N^2}{M_R} \quad (2.57)$$

with M_R given in (2.35).

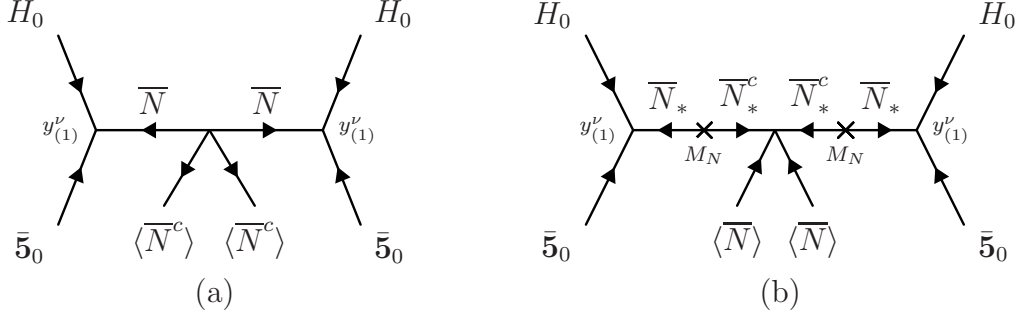


Figure 2: Diagrams which generate $W \ni H(\mathbf{5})H(\mathbf{5})\bar{\mathbf{5}}\bar{\mathbf{5}}$ for the 4+1 model (a) and for the 3+2 model (b).

In the presence of non-vanishing vev's of $\bar{N} \in (\bar{\mathbf{3}}, \mathbf{2}, \mathbf{1})^{-5}$, $(\infty + \infty) \times (\infty + 3 + \infty)$ mass matrix of the $SU(5)_{\text{GUT}}\text{-}\mathbf{10} + \overline{\mathbf{10}}$ sector is deformed into

$$\begin{pmatrix} \mathbf{10}'^c & \mathbf{10}^c_* \end{pmatrix} \left(\frac{M_{\mathbf{10}'}}{y_{(1)}^\nu \langle \bar{N} \rangle} \middle| \begin{array}{c} 0 \\ M_{\mathbf{10}} \end{array} \right) \begin{pmatrix} \mathbf{10}'_* \\ \mathbf{10}_0 \\ \mathbf{10}_* \end{pmatrix}, \quad (2.58)$$

which means that the $U(1)_{\tilde{q}_7}$ eigenstates are related to mass eigenstates as

$$\begin{pmatrix} \mathbf{10}'_* \\ \mathbf{10}_0 \\ \mathbf{10}_* \end{pmatrix} = \begin{pmatrix} 0 & * & * \\ 1 & 0 & 0 \\ 0 & * & * \end{pmatrix} \begin{pmatrix} \hat{\mathbf{10}}_0 \\ \hat{\mathbf{10}}_* \\ \hat{\mathbf{10}}_* \end{pmatrix}. \quad (2.59)$$

Mass matrix in the $\bar{\mathbf{5}} + \mathbf{5}$ sector becomes

$$\begin{pmatrix} \bar{H}^c_* & \bar{5}_0^c & \bar{5}_*^c & H_0 & H_* \end{pmatrix} \left(\begin{array}{cc|cc|c} & & M_{\bar{H}} & & \\ y_{(2)}^\nu \langle \bar{N} \rangle & y_{(2)}^\nu \langle \bar{N} \rangle & & & \\ y_{(2)}^\nu \langle \bar{N} \rangle & y_{(2)}^\nu \langle \bar{N} \rangle & & & \\ & & M_{\bar{5}} & & \\ & & & & \\ y_{(1)}^\nu \langle \bar{N} \rangle & y_{(1)}^\nu \langle \bar{N} \rangle & & & \\ y_{(1)}^\nu \langle \bar{N} \rangle & y_{(1)}^\nu \langle \bar{N} \rangle & & M_H & \end{array} \right) \begin{pmatrix} \bar{H}_0 \\ \bar{H}_* \\ \bar{5}_0 \\ \bar{5}_* \\ H_*^c \end{pmatrix}, \quad (2.60)$$

and the $U(1)_{\tilde{q}_7}$ eigenstates have massless eigenstates as components specified by

$$\begin{pmatrix} \bar{H}_0 \\ \bar{H}_* \\ \bar{\mathbf{5}}_0 \\ \bar{\mathbf{5}}_* \\ H_*^c \end{pmatrix} \sim \begin{pmatrix} \frac{M_{\bar{\mathbf{5}}}}{\sqrt{M_{\bar{\mathbf{5}}}^2 + |y_{(2)}^\nu \langle \bar{N} \rangle|^2}} & 0 & * & * & * \\ 0 & 0 & * & * & * \\ 0 & \frac{M_H}{\sqrt{|M_H|^2 + |y_{(1)}^\nu \langle \bar{N} \rangle|^2}} & * & * & * \\ -\frac{y_{(2)}^\nu \langle \bar{N} \rangle}{\sqrt{M_{\bar{\mathbf{5}}}^2 + |y_{(2)}^\nu \langle \bar{N} \rangle|^2}} & 0 & * & * & * \\ \mathcal{O}\left(\frac{\langle \bar{N} \rangle^2}{M_{\bar{\mathbf{5}}} M_H}\right) & -\frac{y_{(1)}^\nu \langle \bar{N} \rangle}{\sqrt{|M_H|^2 + |y_{(1)}^\nu \langle \bar{N} \rangle|^2}} & * & * & * \end{pmatrix} \begin{pmatrix} \hat{\bar{H}}_0 \\ \hat{\bar{\mathbf{5}}}_0 \\ \hat{\bar{\mathbf{5}}}_* \\ \hat{\bar{\mathbf{5}}}_* \\ \hat{\bar{\mathbf{5}}}_* \end{pmatrix}. \quad (2.61)$$

Using (2.59) and (2.61) in (2.48), one finds that the trilinear R-parity violating operators (1.1) involving massless eigenstates are not generated. The essence is that $U(1)_{\tilde{q}_7}$ -eigenstates $\bar{H}(\bar{\mathbf{5}})$ do not have the massless eigenstates $\hat{\bar{\mathbf{5}}}_0$ as a component, and $U(1)_{\tilde{q}_7}$ -eigenstates $\mathbf{10}'$ do not contain massless eigenstates $\hat{\mathbf{10}}_0$.²¹ Eigenvectors for massless eigenstates in (2.59, 2.61) have non-vanishing entries only when holomorphic insertion of $U(1)_{\tilde{q}_7}$ -breaking vev of \bar{N} can fill the gap of $U(1)_{\tilde{q}_7}$ -charges, just like in the 4+1 model. Therefore, trilinear interactions involving only massless eigenstates are subject to the selection rule based on holomorphic insertion of $\langle \bar{N} \rangle$ and $U(1)_{\tilde{q}_7}$ -charge counting.

In the doublet part²² of $SU(5)_{\text{GUT}}\text{-}\mathbf{5} + \bar{\mathbf{5}}$, one $H_u \subset \hat{H}_0$, one $H_d \subset \hat{H}_0$, and three $L_i \subset \hat{\bar{\mathbf{5}}}_0$ should remain in the low-energy spectrum, although H_u and $\{H_d, L_i\}$ are in a pair of vector-like representations of the MSSM gauge group. Thus, the mass matrix (2.60) somehow has to have a reduced rank for the doublet part. Note that it is actually an $((\infty + a + \infty + (1 + b) + \infty) \times ((1 + a) + \infty + (3 + b) + \infty + \infty))$ matrix for the doublet part,²³ although it looks like 5×5 ; here a and b denote the number of massless modes in $\bar{\mathbf{5}}^c\text{-}\bar{H}$ and $H\text{-}\bar{\mathbf{5}}$ vector-like pairs, respectively, that become massive after spontaneous $U(1)_{\tilde{q}_7}$ breaking. It is supposed to have rank $\infty + a + \infty + b + \infty$, not $\infty + a + \infty + (1 + b) + \infty$.

First of all, in order for three L_i 's to remain in the low-energy spectrum, the $(1 + b) \times (3 + b)$ submatrix—(4th, 3rd) block—of (2.60) should have rank b , not $(1 + b)$. Such rank reduction

²¹ The 3+2 model corresponds to a rank-5 vector bundle V given by an extension of a rank-3 bundle U_3 by a rank-2 bundle U_2 :

$$0 \rightarrow U_2 \rightarrow V \rightarrow U_3 \rightarrow 0. \quad (2.62)$$

U_2 is a sub-bundle of V , and $U_2 \otimes V$ that of $\wedge^2 V$. The massless eigenstates $\hat{\mathbf{10}}_0$ and $\hat{\bar{\mathbf{5}}}_0$ are characterized as $H^1(X; U_2) \subset H^1(X; V)$ and $H^1(X; U_2 \otimes V) \subset H^1(X; \wedge^2 V)$, respectively, and have restricted interactions than general massless states in $H^1(X; V)$ and $H^1(X; \wedge^2 V)$, respectively.

²² See footnotes 6 and 17 for discussions of doublet-triplet splitting.

²³ For the triplet part, $(1 + b)$ and $(1 + a)$ are replaced by b and a , respectively. Rank reduction is not necessary.

of mass matrix can happen for a generic point on the gauge-field moduli space, as we have reviewed in section 2.1, and remarked as the first possibility for the light two Higgs doublets in the 4+1 model. This means that there are one linear combination of H_0 's and three of $\bar{\mathbf{5}}_0$'s that do not have couplings of the form $W \ni y^\nu H_0 \bar{N}_0 \bar{\mathbf{5}}_0$.

Secondly, another massless doublet $H_d \subset \hat{H}_0$ also has to remain in the low-energy spectrum. In general, \hat{H}_0 -type zero modes and $\hat{\bar{H}}_0$ -type zero modes effectively have mass terms of the form $W \ni [(y_{(1)}^\nu \langle \bar{N}_0 \rangle)(y_{(2)}^\nu \langle \bar{N}_0 \rangle)/M_{\mathbf{5}}] \hat{H}_0 \hat{\bar{H}}_0$, and become massive. It may be, however, that this mass matrix has a reduced rank for generic moduli field (including \bar{N} 's) value, just like we assumed above. Alternatively, the effective mass parameter happens to be small at the value of $\langle \bar{N} \rangle$'s where moduli are stabilized. While the first possibility is aesthetically better, the origin of μ parameter is not explained because the zero modes \hat{H}_0 and $\hat{\bar{H}}_0$ are not coupled to moduli fields that might play a role of the singlet field of the NMSSM. The second scenario seems to involve a fine-tuning to get the μ parameter small;²⁴ some anthropic selection may be responsible for it [20].

Remark

There are some other variations of the 4+1 and 3+2 models [7], but they are essentially the same in the mechanism of eliminating the trilinear R-parity violating operators. Thus, we use these two models as illustrative examples representing a theoretical framework that can be an alternative to R parity.

2.3 Dimension-5 Proton Decay Operator

The idea in section 2.2 was that the mixing of massless modes is governed by holomorphic insertion of vacuum expectation value of chiral multiplets. The trilinear R-parity violating operators are absent and the dimension-4 proton decay problem is solved, without an unbroken discrete symmetry such as R-parity. Supersymmetric extensions of the standard model, however, has another problem. If the effective superpotential contains

$$\Delta W = \frac{1}{M_1} Q Q Q L + \frac{1}{M_2} \bar{E} \bar{U} \bar{U} \bar{D} \equiv \mathcal{O}_1 + \mathcal{O}_2 \subset \frac{1}{M_{\text{eff}}} \mathbf{101010\bar{5}}, \quad (2.63)$$

these terms violate both baryon number and lepton number, and hence proton decays. Thus, we discuss in this subsection whether or not the theoretical framework in section 2.2 predicts the operators above in the low-energy effective superpotential.

²⁴There may be multiple \bar{N} 's that acquire vev's, and there are infinitely many Kaluza-Klein states $\bar{\mathbf{5}}_*^c - \bar{\mathbf{5}}_*$. Thus, the effective μ -parameter being small does not necessarily mean that $y^\nu \langle \bar{N} \rangle$ itself is small.

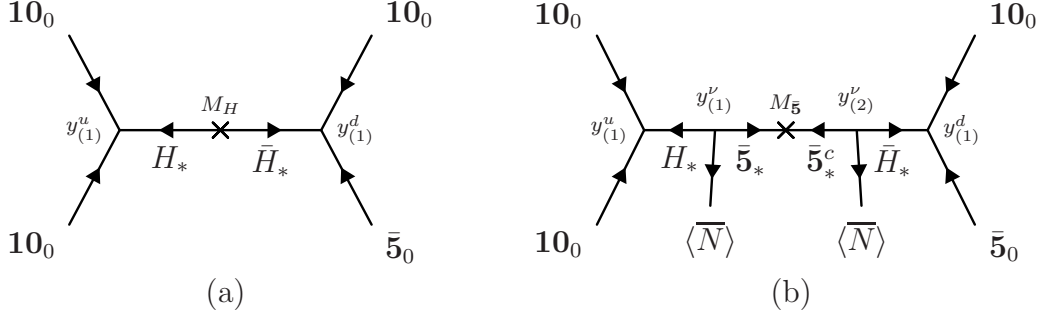


Figure 3: Dimension-5 proton decay operators in $W \ni \mathbf{101010\bar{5}}$ are generated by a diagram (a) in the 4+1 model. Although they appear to be generated in the 3+2 model as well through the diagram in (b), it turns out that they are actually not. See the text for explanation.

4+1 model

In the minimal supersymmetric SU(5) GUT model, the dimension-5 operators above are generated by integrating out colored Higgs multiplets. The essence is that the pair of colored Higgs multiplets in the $SU(3)_C\text{-}\mathbf{3} + \bar{\mathbf{3}}$ representations has a vector-like mass term. The 4+1 model in section 2.2 also shares the same property. The up-type and down-type Higgs doublets of the MSSM originate from irreducible components H and \bar{H} in the $SU(5)_{\text{GUT}}\text{-}\mathbf{5} + \bar{\mathbf{5}}$ representations, and they form a Hermitian conjugate pair even in the E_8 gauge group. Therefore, the superdiagram in Figure 3 (a) generates the term $\Delta W = \mathbf{101010\bar{5}}$ in low-energy effective theory, similarly to the minimal supersymmetric SU(5) GUT model. In particular, the $QQQL + \bar{E}\bar{U}\bar{U}\bar{D}$ part relevant to proton decay is generated by colored Higgsino exchange.²⁵

There are a couple of differences, however, between the minimal supersymmetric SU(5) GUT model and the 4+1 model. In the 4+1 model, propagating between the two vertices of Figure 3 (a) are not just one pair of colored Higgs multiplets. All of massive Kaluza-Klein excitations contribute to the amplitude. These infinite number of contributions add up coherently. Thus, we have an effective operator

$$W_{\text{eff.}} \ni \sum_I \frac{y_I^u y_I^d}{M_{H;I}} \mathbf{10}_0 \mathbf{10}_0 \mathbf{10}_0 \bar{\mathbf{5}}_0 \equiv \frac{1}{M_{\text{eff.}}} \mathbf{10}_0 \mathbf{10}_0 \mathbf{10}_0 \bar{\mathbf{5}}_0, \quad (2.64)$$

with massless eigenstates in the external lines. As this effective operator is generated in the 4+1 model without a mixing induced by $\langle \bar{N}^c \rangle$, we do not make an explicit distinction between the $U(1)_\chi$ -eigenstates and mass-eigenstates. Yukawa couplings y_I are determined by overlap

²⁵Scalar colored Higgs exchange is also relevant to $QQQL + \bar{E}\bar{U}\bar{U}\bar{D}$ proton decay.

integration of mode functions of three relevant states;

$$y_I = \frac{g_{\text{YM}}}{\text{vol}(X)} \int_X d^6 y \, \text{tr} (\bar{\chi}_0 \gamma^m \varphi_{0;m} \chi_I), \quad (2.65)$$

where χ_0 and $\varphi_{0;m}$ are wavefunctions of massless modes to be used in the external lines, and χ_I that of I -th massive Higgs multiplet in the internal line with mass $M_{H,I}$. The overall factor²⁶ g_{YM} originates from the normalization convention of the mode functions in (2.2).

Sum of infinite contributions in (2.64) is treated better in an equivalent description

$$\frac{1}{M_{\text{eff}}} = \frac{g_{\text{YM}}}{\text{vol}(X)} \int_X d^6 y (\bar{\chi}_0 \gamma^m \varphi_{0;m})(y) \frac{g_{\text{YM}}}{\text{vol}(X)} \int_X d^6 y' (\varphi_{0;n} \gamma^n \chi_0)(y') \sum_I \frac{\chi_I(y) \bar{\chi}_I(y')}{M_{H,I}} \quad (2.66)$$

$$\sim \frac{g_{\text{YM}}^2}{\text{vol}(X)} \int_X d^6 y (\bar{\chi}_0 \gamma^m \varphi_{0;m})(y) \left[\int \frac{d^6 p'}{(2\pi)^6} \frac{e^{ip \cdot (y-y')}}{p} \right] \int_X d^6 y' (\varphi_{0;n} \gamma^n \chi_0)(y'), \quad (2.67)$$

Discrete summation labeled by I in the first line is approximated by continuous momentum integration in the second line. This is a reasonable thing to do, as long as we are interested in contributions from highly excited Kaluza–Klein states. The factor in the square bracket in (2.67) is nothing but Green function over the internal space X , and its short-distance singularity $\gamma^k \partial_k (1/|y - y'|^4)$ corresponds to summing up contributions from infinitely many Kaluza–Klein states. For zero-mode wavefunctions χ_0 and $\varphi_{0;m}$ without particularly singular or rapidly varying behavior, integration over $|y - y'|$ in (2.67) is dominated by the long-distance region, $|y - y'| \approx R$, not by the short-distance region. Thus, $1/M_{\text{eff}}$ is finite. We also learn here that contributions from low-lying triplet Higgsino exchange dominate the amplitude $1/M_{\text{eff}}$ of dimension-5 proton decay operators.

Experimental limits on the dimension-5 operators (2.63) are set by several decay modes of proton. $\tau(p \rightarrow K^+ + \bar{\nu}) \gtrsim 2.3 \times 10^{33} \text{ yrs.}$ [23], one of the most important ones in the minimal supersymmetric $\text{SU}(5)_{\text{GUT}}$ model, roughly corresponds to

$$M_{\text{eff.}} \gtrsim 10^{24} \text{ GeV.} \quad (2.68)$$

Limits from other decay modes are somewhat different, but not by several orders of magnitude. All kinds of GUT models are marginally in conflict with this constraint [24, 25] as long as the two Higgs triplets are vector like in any underlying symmetry groups. This property is shared also by the 4+1 model²⁷ If $1/M_{\text{eff.}}$ is approximated by contributions from a few lightest triplet

²⁶ Mode functions should be normalized so that the kinetic terms of chiral multiplets are normalized canonically. The expression (2.65) contains $e^{\langle K \rangle / M_G^2}$, where $\langle K \rangle / M_G^2$ is a vev of (2.10).

²⁷The R-parity preserving scenario explained at the end of section 2.1. also does.

massive Higgsinos in the Kaluza–Klein tower, $1/M_{\text{eff.}} \approx y_I^u y_I^d / M_{\text{KK}}$, the experimental limit is translated to

$$\sqrt{y_I^u y_I^d} \lesssim 10^{-4} \times \left(\frac{M_{\text{KK}}}{10^{16} \text{ GeV}} \right)^{1/2}. \quad (2.69)$$

This constraint seems a little severe if the Kaluza–Klein scale M_{KK} is around the GUT scale $M_{\text{GUT}} \sim 10^{16} \text{ GeV}$, and the trilinear couplings are of the order of the Yukawa couplings of the second generation quarks, $y_c \sim 10^{-2}$ and $y_s \sim 10^{-3}$. However, $y_I^{u,d}$ s in (2.64, 2.69) in the 4+1 model are not related to the Yukawa couplings of quarks and leptons by $\text{SU}(5)_{\text{GUT}}$ symmetry. y_I ’s are calculated in (2.65) by using mode functions of Kaluza–Klein triplet Higgsinos $\chi_I(y)$, whereas the observable Yukawa couplings are calculated by using mode functions of two massless doublet Higgsinos. Since y_I ’s are “Fourier transform” of zero-mode wavefunctions $(\bar{\chi}_0 \gamma^m \varphi_{0;m})$ on a compact internal manifold X , they become very small for higher Kaluza–Klein states. Therefore, it will not be conservative to exclude the 4+1 model with $M_{\text{KK}} \approx M_{\text{GUT}}$.

Remark

There is a theoretical side remark here, before moving over to the 3+2 model. In the process going from (2.9) and/or (2.20–2.27) to (2.64), we are rewriting 1PI effective action: from the one written in terms of (Kaluza–Klein decomposition of) Yang–Mills multiplets on 9+1 dimensions to the one in terms only of Kaluza–Klein zero modes on 3+1 dimensions. The dimension-5 operator (2.64) is *necessary* as a 1PI vertex when 1PI effective action is written in terms only of zero modes, although it was *absent* in (2.9) where all Kaluza–Klein modes appear in 1PI effective action. It is therefore almost trivial that the sum of infinite contributions in (2.64) is finite; the amplitude described by (2.64) and by Figure 3 (a) is nothing more than a *tree-level* scattering amplitude of super Yang–Mills theory with the external states having wavepackets of Kaluza–Klein zero modes.

3+2 model

Let us now study whether the dimension-5 proton decay operators (2.63) are generated in the 3+2 model. In this model, $\text{U}(1)_{\tilde{q}\tau}$ -eigenstates H and \bar{H} are not Hermitian conjugate, and hence the dimension-5 proton decay operators are absent in the effective theory in the $\langle \bar{N} \rangle = 0$ limit. Thus, whether those operators exist in low-energy effective theory depends on how the vev’s of \bar{N} can be inserted in the effective superpotential.

$\text{U}(1)_{\tilde{q}\tau}$ -breaking vev’s $\langle \bar{N} \rangle$ induce mixing between states with different $\text{U}(1)_{\tilde{q}\tau}$ charges. The mixing can take place among massive states propagating in the internal line as in the diagram in Figure 3 (b). On the other hand, the massless eigenstates $\hat{\mathbf{1}}_0$ in the external lines remain pure $\text{U}(1)_{\tilde{q}\tau}$ -eigenstates, as we saw in (2.59). Although H_*^c -type $\text{U}(1)_{\tilde{q}\tau}$ -eigenstates contain the

massless eigenstates $\hat{\mathbf{5}}_0$ as components, there is no interaction involving $\mathbf{10}$ and H^c simultaneously in (2.48). Thus, mixing in the internal line is responsible, if the dimension-5 proton decay operators are to be generated.

Let M_{IJ} denote the mass matrix in (2.60), and U and V unitary matrices that diagonalize M_{IJ} ;

$$U_{HI} M_{IJ} V_{JK} = \delta_{HK} \hat{M}_K. \quad (2.70)$$

Then the dimension-5 proton decay operators in the effective theory are given by

$$W_{\text{eff.}} \ni \sum_{I \in H(\mathbf{5}), J \in \bar{H}(\bar{\mathbf{5}})} \sum_K' \frac{(y_J^d V_{JK})(y_I^u U_{KI})}{\hat{M}_K} \mathbf{10}_0 \mathbf{10}_0 \mathbf{10}_0 \hat{\mathbf{5}}_0 \equiv \frac{1}{M_{\text{eff.}}} \mathbf{10}_0 \mathbf{10}_0 \mathbf{10}_0 \hat{\mathbf{5}}_0; \quad (2.71)$$

the prime in the summation means that the label K runs only over massive states. The effective coupling $1/M_{\text{eff.}}$ can be expressed in a much simpler way.

$$\frac{1}{M_{\text{eff.}}} = y_{(1)}^d \left(\sum_K V_{\bar{H}(\bar{\mathbf{5}})K} \hat{M}_K^{-1} U_{KH(\mathbf{5})} \right) y_{(1)}^u = y_{(1)}^d (M^{-1})_{\bar{H}(\bar{\mathbf{5}})H(\mathbf{5})} y_{(1)}^u. \quad (2.72)$$

Thus, although the unitary mixing matrices U and V are not given by rational functions of parameters in the superpotential, the coupling $1/M_{\text{eff.}}$ is in the effective superpotential.

Let us examine the matrix M^{-1} , first with the simplest case $a = b = 0$ in Table 3. This corresponds to a case where there is no pair of chiral multiplets in the $\text{SU}(5)_{\text{GUT}}\text{-}\mathbf{\bar{5}} + \mathbf{5}$ representations that acquire masses after $\langle \bar{N} \rangle$'s become non-zero. First, we are concerned only about the $\mathbf{3} + \bar{\mathbf{3}}$ part of the mass matrix in the $\mathbf{5} + \bar{\mathbf{5}}$ sector, because the dimension-5 proton decay operators are generated only through triplet exchange diagrams. Second, three massless chiral multiplets $\bar{D}_i \subset \hat{\mathbf{5}}_0$ are taken out of the triplet part of the mass matrix (2.60), because only massive states are integrated out. Now, the mass matrix (2.60) has become

$$M_{IJ} = \begin{pmatrix} M_{\bar{H}} & 0 & 0 \\ y_{(2)}^\nu \langle \bar{N} \rangle & M_{\bar{\mathbf{5}}} & 0 \\ 0 & y_{(1)}^\nu \langle \bar{N} \rangle & M_H \end{pmatrix}, \quad (2.73)$$

and we treat it as if it were a 3×3 matrix. The index I runs over $\{\bar{H}^c(\mathbf{3}), \bar{\mathbf{3}}^c, H(\mathbf{3})\} \subset \{\bar{H}^c(\mathbf{5}), \bar{\mathbf{5}}^c, H(\mathbf{5})\}$, and J over $\{\bar{H}(\bar{\mathbf{3}}), \bar{\mathbf{3}}, H^c(\bar{\mathbf{3}})\} \subset \{\bar{H}(\bar{\mathbf{5}}), \bar{\mathbf{5}}, H^c(\bar{\mathbf{5}})\}$. There is no qualitative problem in approximating the summation over contributions from infinite Kaluza–Klein states by a sum over three distinct Kaluza–Klein towers, as we have discussed before for the 4+1 model. The entire contribution from a tower is finite, and only those from low-lying massive states have sizable contributions. It is now easy to see that the cofactor of $(H(\mathbf{3}), \bar{H}(\bar{\mathbf{3}}))$ element

of the matrix (2.73) vanishes, and so is $(M^{-1})_{\bar{H}(\bar{\mathbf{3}})H(\mathbf{3})}$. Thus, the dimension-5 proton decay operators are not generated in the 3+2 model with spectra characterized by $a = b = 0$.

Individual contribution from a given mass-eigenstate in (2.71), $V_{JK}\hat{M}_K^{-1}U_{KI}$ without summation over K , does not vanish, as the Feynman diagram in Figure 3 (b) indicates. Some of those contributions are of order $M(y^\nu \langle \bar{N} \rangle)^{*2}/|M|^4$, where $M \approx M_{\bar{H},\bar{\mathbf{5}},H}$. But, all these contributions cancel, and the total amplitude $1/M_{\text{eff.}}$ vanishes. The total $1/M_{\text{eff.}}$ should be given by a rational function of parameters in the original superpotential, as we saw in (2.72). Vev's of anti-chiral multiplets $(y^\nu \langle \bar{N} \rangle)^{*2}$ should not survive cancellation.

Similar study can be carried out for cases with either/both a or/and b is/are non-zero.²⁸ The 5×5 matrix (2.60) without rank reduction is used instead of (2.73) for the case with $a \neq 0$ and $b \neq 0$, and 4×4 matrices are used for the two other cases. The cofactor of the $(H(\mathbf{3}), \bar{H}(\bar{\mathbf{3}}))$ element turn out to vanish for all these matrices, and hence the dimension-5 proton decay operators are not generated for any of these cases. Thus, the dimension-5 proton decay operators are absent in low-energy effective superpotential of the 3+2 model, independent of whether $\bar{\mathbf{5}}^c\text{--}\bar{H}(\bar{\mathbf{5}})$ like and/or $H(\mathbf{5})\text{--}\bar{\mathbf{5}}$ like states with masses of order $y^\nu \langle \bar{N} \rangle$ exist at high energy or not.

The absence of the operator in the 3+2 model can be understood in terms of $U(1)_{\tilde{q}_7}$ -charge counting. Requiring that the effective coupling $1/M_{\text{eff.}}$ is a rational function of parameters and vev's of the original superpotential, and that the effective superpotential is invariant under the $U(1)_{\tilde{q}_7}$ symmetry, the only possible form is $1/M_{\text{eff.}} \sim y^u y^d M / (y^\nu \langle \bar{N} \rangle)^2$, where M is some mass scale. Unless there are massive states in the $SU(5)_{\text{GUT}}\text{--}\bar{\mathbf{5}} + \mathbf{5}$ representations with their masses given by $\approx \mathcal{O}((y^\nu \langle \bar{N} \rangle)^2/M)$, such terms are not generated in the low-energy effective superpotential. This could have been the case if there were extra pairs of zero modes of $H(\mathbf{5})$ -type and $\bar{H}(\bar{\mathbf{5}})$ -type, in addition to those listed in Table 3. We have ignored such a possibility so far, because the dimension-5 proton decay operators with $1/M_{\text{eff.}} \sim y^u y^d M_{\bar{\mathbf{5}}} / (y^\nu \langle \bar{N} \rangle)^2$ seem too large compared with the experimental limits, if $M_{\bar{\mathbf{5}}} \sim M_{\text{KK}} \sim M_{\text{GUT}}$ and $y^\nu \langle \bar{N} \rangle \ll M_{\text{KK}}$, despite large uncertainties associated with trilinear couplings y^u and y^d that involve Kaluza-Klein states.

2.4 Bilinear R-parity Violation

Let us now study consequences of R-parity violation in the framework presented in section 2.2. We begin with a study of mixing between $L_i \subset \hat{\bar{\mathbf{5}}}_0$ and $H_d \subset \hat{\bar{H}}(\bar{\mathbf{5}})_0$; they are in the

²⁸In these cases, extra massless multiplets in the $SU(5)_{\text{GUT}}\text{--}\bar{\mathbf{5}} + \mathbf{5}$ representations appear in the $\langle \bar{N} \rangle \rightarrow 0$ limit.

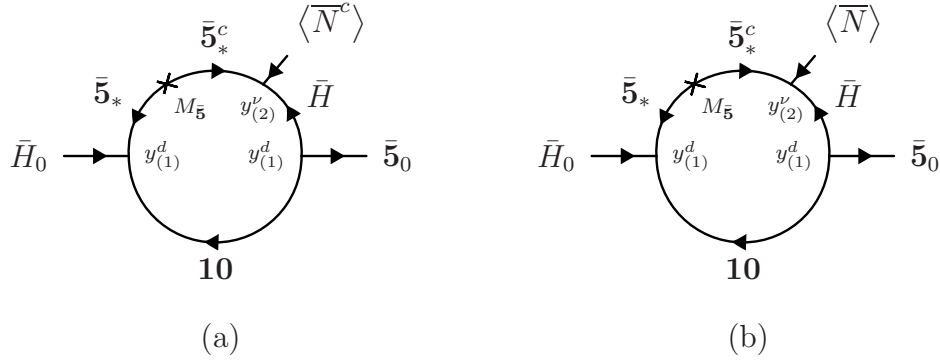


Figure 4: Diagrams which produce $K \ni \bar{\mathbf{5}}^\dagger \bar{H}$ for the 4+1 model (a) and for the 3+2 model (b).

same representation of the MSSM gauge group $SU(2)_L \times U(1)_Y \subset SU(5)_{\text{GUT}}$, and have distinct symmetry charges only in the presence of an unbroken R-parity or $U(1)$ symmetry. Any interactions involving R-parity violation (and hence a vev of \overline{N}^c (in the 4+1 model) or \overline{N} (in the 3+2 model)) have a chance to induce mixing between L_i 's and H_d .

We have already seen in section 2.2 that the vev's of \overline{N}^c [resp. \overline{N}] deform mass matrices in the superpotential in the 4+1 [resp. 3+2] model. Consequences of the deformed mass matrices, however, were quite limited in the superpotential at the renormalizable level. Mixing of massless eigenstates are under strict control of the spontaneously broken $U(1)_\chi$ [resp. $U(1)_{\tilde{q}_7}$] symmetry, and the trilinear R-parity violating operators (1.1) are not generated for massless modes. Although the $U(1)$ symmetry does not rule out the other trilinear R-parity violating operator of the form (2.41), it is absent in the MSSM because there is only one down-type Higgs doublet. Yukawa couplings of quarks and leptons will have different values from those in (2.20–2.24) [resp. (2.46–2.50)], because the massless eigenstates may not be exactly the same as the original $U(1)$ -eigen zero modes. Those are all the consequences.

Vev's of \overline{N}^c or \overline{N} , however, generate mixing in the *Kähler potential* as well. Kinetic mixing

$$K_{\text{eff.}} \ni \epsilon \bar{\mathbf{5}}^\dagger \bar{H}(\bar{\mathbf{5}}) + \text{h.c.} \quad (2.74)$$

is generated in both the 4+1 and 3+2 model at 1-loop by super Feynman diagrams in Figure 4. Those diagrams give rise to operators,

$$\text{4+1 model} \quad K_{\text{eff.}} \sim \frac{|y^d|^2}{16\pi^2} \frac{M_{\bar{\mathbf{5}}}^* y'^\nu \langle \overline{N}^c \rangle}{|M|^2} \bar{\mathbf{5}}^\dagger \bar{H} + \text{h.c.}, \quad (2.75)$$

$$\text{3+2 model} \quad K_{\text{eff.}} \sim \frac{|y^d|^2}{16\pi^2} \frac{M_{\bar{\mathbf{5}}}^* y^\nu \langle \overline{N} \rangle}{|M|^2} \bar{\mathbf{5}}^\dagger \bar{H} + \text{h.c.} \quad (2.76)$$

$|M|^2$'s in the denominators stand for the largest among $|M_{\bar{\mathbf{5}}}|^2$, $|M_{\mathbf{10}}|^2$ and $|M_{\bar{H}}|^2$, because that is where the dominant contribution comes from in loop momentum integration. There are two other kinds of contributions as well, but they are quite similar to those above; see the caption of Figure 1. We treat 1-loop amplitudes here, as if only finite number of massive particles ran in the loop.

It is true in the context of string compactification that infinitely many Kaluza–Klein particles and stringy states are also running in the loop. The argument above estimates contributions only from low-lying Kaluza–Klein multiplets. Unless the remaining UV contributions exactly cancels the IR contributions above, however, the total 1-loop amplitude does not vanish. Since the IR and UV contributions are likely to depend on geometry differently, it is unlikely that they cancel. Thus, as long as there are non-vanishing IR contributions, it is likely that the total amplitude does not vanish. This is what we can guess from the argument using Feynman diagrams.

Although the UV contributions to ϵ have not been discussed yet, it is sufficient to treat it symbolically in seeing that its effects disappear from renormalizable interactions of the massless modes of the MSSM. By a non-unitary basis transformation

$$\begin{pmatrix} \bar{H} \\ \bar{\mathbf{5}} \end{pmatrix} = \begin{pmatrix} 1 & \\ -\epsilon & 1 \end{pmatrix} \begin{pmatrix} \bar{H}' \\ \bar{\mathbf{5}}' \end{pmatrix}, \quad (2.77)$$

the kinetic terms are diagonalized, yet the R-parity violating trilinear operators (1.1) are absent when the superpotential is rewritten in terms of newly defined chiral multiplets \bar{H}' and $\bar{\mathbf{5}}'$. Although operators of the form (2.41) now exist when written in the new chiral multiplets, there is no such term that consists only of massless multiplets in the MSSM because of anti-symmetric contraction of $\text{SU}(5)_{\text{GUT}}$ indices [8]. Jacobian of this field redefinition in path integral is trivial, and there are no other effects from this redefinition.

Similarly, 1-loop amplitudes generate kinetic mixing among chiral multiplets in the $\text{SU}(5)_{\text{GUT}} - \mathbf{10} + \overline{\mathbf{10}}$ representations. An appropriate field redefinition, however, can diagonalize the kinetic terms without generating R-parity violating trilinear operators that involve only massless multiplets in the MSSM. In the 3+2 model, for example, an effective Kähler potential term

$$K_{\text{eff.}}^{(3+2)} \sim \epsilon \mathbf{10}^\dagger \mathbf{10}' + \text{h.c.} \quad (2.78)$$

is generated, with a coefficient ϵ proportional to $y^\nu \langle \overline{N} \rangle$. A new basis $(\mathbf{10}', \mathbf{10})'$ is chosen by

$$\begin{pmatrix} \mathbf{10}' \\ \mathbf{10} \end{pmatrix} = \begin{pmatrix} 1 & \\ -\epsilon & 1 \end{pmatrix} \begin{pmatrix} \mathbf{10}' \\ \mathbf{10} \end{pmatrix}', \quad (2.79)$$

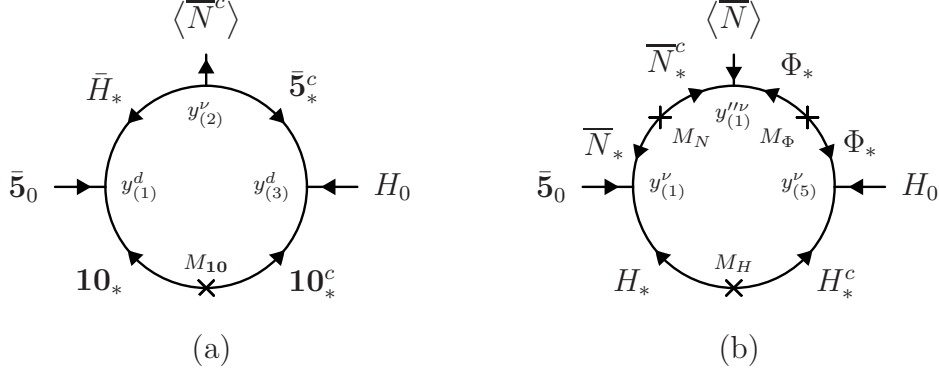


Figure 5: (a) is a typical diagram for (2.81) in 4+1 model. (b) is a typical diagram for (2.82) in 3+2 model.

so that the kinetic terms become diagonal. Note that the redefinitions of chiral multiplets (2.77) and (2.79) use parameters ϵ , which involve only holomorphic vev's of either \bar{N}^c or \bar{N} . This is why the basis transformation matrices (2.77) and (2.79) are lower triangular. It thus follows that the mass matrices (2.38, 2.60) and (2.58) remain lower triangular, and whole argument we have had so far in sections 2.2 and 2.3 does not have to be changed qualitatively.²⁹

The Kähler potential of the effective theory may also have a bilinear term

$$K_{\text{eff.}} \ni c \bar{5} H(\bar{5}) + \text{h.c.} \quad (2.80)$$

1-loop super Feynman diagrams in Figure 5 generate

$$\text{4+1 model} \quad K_{\text{eff.}} \ni \frac{y^{d2}}{16\pi^2} \frac{(y^\nu \langle \bar{N}^c \rangle)^* M_{10}^*}{|M|^2} \bar{5} H + \text{h.c.}, \quad (2.81)$$

$$\text{3+2 model} \quad K_{\text{eff.}} \ni \frac{y^{\nu 2}}{16\pi^2} \frac{(y^\nu \langle \bar{N} \rangle) M_N^* M_\Phi^* M_H^*}{|M|^4} \bar{5} H + \text{h.c.}, \quad (2.82)$$

where $|M|$'s in the denominator are the scale where the dominant contribution of the loop momentum integration comes from field theory on 3+1 dimensions. Similar contributions come from two other kinds of diagrams with particles in different $\text{SU}(5)_{\text{GUT}}$ representations running in the loop, just like several kinds of diagrams in Figure 1. In both models, the coefficient c in (2.80) is roughly of the form

$$c \sim \frac{y^2}{16\pi^2} \frac{y \langle N \rangle}{M}, \quad (2.83)$$

²⁹We will make an order of magnitude estimate of ϵ at the end of this section, and argue that the additional deformation to mass matrices is quantitatively unimportant compared with the tree-level deformation discussed in section 2.2.

where phases are ignored, and $\langle N \rangle$ means $\langle \bar{N}^c \rangle$ in the 4+1 model and $\langle \bar{N} \rangle$ in the 3+2 model. The estimate (2.83) takes account only of 1-loop amplitudes with low-lying Kaluza–Klein multiplets in the loop, and there will also be UV contributions where higher Kaluza–Klein states and stringy states are in the loop. Although we have not estimated the UV contributions, it is unlikely that the UV and IR contributions cancel almost exactly, as we discussed for the kinetic mixing $K \ni \bar{\mathbf{5}}^\dagger \bar{H}$. Thus, it is likely that c does not vanish in low-energy effective theory.

Holomorphic mass parameters M are expected to have θ^2 component in a vacuum with broken supersymmetry, at least of order $M(1 + \theta^2 m_{3/2})$. Therefore, the anti-holomorphic mass parameters in (2.81, 2.82) have $\bar{\theta}^2$ components, and hence the 1-loop amplitudes generate an R-parity violating bilinear term

$$W_{\text{eff.}} \ni c m_{3/2} \bar{\mathbf{5}} H(\mathbf{5}). \quad (2.84)$$

In the $\text{SU}(3)_C$ triplet part, this additional mass term $\Delta W = \mu_i \bar{D}_i H(\mathbf{3})_*$ is not significant. It deforms the mass matrix including $W \ni M_H H^c(\bar{\mathbf{3}})_* H(\mathbf{3})_*$ in the $\text{SU}(3)_C - \bar{\mathbf{3}} + \mathbf{3}$ sector, and induces mixing between $H^c(\bar{\mathbf{3}})_*$ and \bar{D} . Massless eigenstates $\hat{\bar{D}}_i$ pick up interactions of $H^c(\bar{\mathbf{3}})_* \subset H^c(\bar{\mathbf{5}})_*$ with mixing coefficients $-\mu_i/M_H$. Thus, the massless eigenstates have trilinear interactions

$$W_{\text{eff.}}^{(4+1)} \ni -y_{(1)}^d \frac{\mu_i}{M_H} \hat{\bar{D}}_i \bar{U} \bar{D} - y_{(1)}^d \frac{\mu_i}{M_H} \hat{\bar{D}}_i Q L, \quad (2.85)$$

in the 4+1 model, where $H^c(\bar{\mathbf{5}}) = \bar{H}(\bar{\mathbf{5}})$. When $M_H \sim M_{\text{KK}}$ is around the GUT scale, effective R-parity violating coupling $\lambda'' \sim -y^d \mu_i/M_H$ is very small, and irrelevant to phenomenology except in nucleon decay processes discussed in section 3.4. This coupling is not even generated in the 3+2 model, because H^c is different from \bar{H} .

In the doublet part, however, the additional mass terms,

$$W_{\text{eff.}} \ni c_i m_{3/2} L_i H_u \equiv \mu_i L_i H_u \quad (2.86)$$

with c_i roughly given by (2.83), are important in phenomenology. Since H_u only has a small mass term $\Delta W = \mu_0 H_d H_u$, with the μ -parameter μ_0 of the order of the electroweak scale, the mixing angle of L_i – H_d mixing, $\sim \mu_i/\mu_0$, is not so small as μ_i/M_H . Although this mixing angle is quite smaller than $\mathcal{O}(1)$ because of the suppression factor c in (2.83), it is much larger than μ_i/M_H and can be phenomenologically significant.

In addition to L_i – H_d mixing in massless states, \hat{L}_i have mixing of the order μ_i/M_H with massive states in the doublet part of $H^c(\bar{\mathbf{5}})_*$. In the 4+1 model, this mixing generates trilinear R-parity violating interactions

$$W_{\text{eff.}} \ni -y_{(1)}^d \frac{\mu_i}{M_H} \hat{L}_i \bar{E} L - y_{(1)}^d \frac{\mu_i}{M_H} \hat{L}_i Q \bar{D}, \quad (2.87)$$

since $H^c(\bar{5}) \equiv \bar{H}(\bar{5})$ in the 4+1 model. If $M_H \sim M_{\text{KK}}$ is around the GUT scale, however, effective trilinear R-parity violating couplings $\lambda \sim \lambda' \sim -y_{(1)}^d \mu_i / M_H$ are so small that it is negligible in phenomenology compared with μ_i / μ_0 contribution from (2.86). Thus, even in the 4+1 model, trilinear R-parity violating couplings (2.85, 2.87) are too small to have phenomenological significance, and the bilinear mass term (2.86) is virtually the only phenomenologically relevant R-parity violation at renormalizable level.³⁰

We have seen so far that the trilinear R-parity violating operators (1.1) are either absent [in the 3+2 model] or negligibly small [in the 4+1 model] in this framework. This framework predicts bilinear-dominated R-parity violation at renormalizable level, which was assumed in [10]. The dimension-4 proton decay problem is absent in bilinear-dominated R-parity violation, as baryon number symmetry is preserved (apart from a small breaking in (2.85)). So, it is easier to satisfy other phenomenological constraints as well in bilinear-term domination, comparing to the case of trilinear-term domination; as long as μ_i / μ 's (and bilinear R-parity violating terms in the SUSY-breaking potential) remain sufficiently small, everything is fine. The only question is why μ_i / μ 's are small.

In our framework, coefficients c_i 's are all suppressed by $(y^\nu \langle N \rangle) / M$ as in [6, 9]. $\mu_i = c_i m_{3/2}$ becomes even smaller for smaller gravitino mass [6].³¹ Even if gravitino mass is not very small, c_i 's involve an extra 1-loop factor in (2.83) since those operators are generated at 1-loop. c_i 's may be suppressed further by some ratio of mass parameters that we find in (2.81, 2.82). Therefore, there are many reasons for μ_i to be small in this framework.

As we have made it clear, however, the rough estimate of the 1-loop amplitude (2.83) only accounts for a partial contribution. To see if we can rely on the estimate, let us first discuss if there are not any tree-level contributions at all. Secondly, we also need to see if the infinite Kaluza–Klein particles and stringy states in the 1-loop amplitude could give significantly larger contributions than (2.83).

In order to find out whether there is a tree-level contribution, it is desirable, in principle, to calculate a sphere amplitude on a Calabi–Yau background. In reality, though, such a calculation is rarely available except in orbifold limits of Calabi–Yau 3-folds. However, tree-level 1PI effective action has been calculated for the Heterotic $E_8 \times E'_8$ string theory on a flat 9+1 dimensional spacetime in α' -expansion. Here, $\alpha' = 1/M_s^2$, and M_s is the string scale. If $\mu_i L_i H_u$ mass terms are obtained from dimensional reduction of the tree-level 1PI effective action on

³⁰In section 3.4, however, we will see that the operator (2.85) can be important for some choice of parameters.

³¹Possible contributions from messenger sector fields in gauge mediation models have to be studied separately. This issue is not covered in this article, since such contributions will depend very much on details of models of gauge mediation.

9+1 dimensions, it is likely that they are obtained also in sphere amplitudes for compactified models. If μ_i 's obtained from dimensional reduction are proportional to a positive power of M_s , it is likely that they are, too, in compactified models, because we expect that the process of compactification introduces only M_{KK} -dependence, not extra M_s dependence. Thus, we content ourselves with guessing whether μ_i 's are generated at tree-level, and if they are generated, how they depend on M_s , by using dimensional reduction of 1PI effective action on the flat 9+1 dimensional spacetime.

In order to find out whether dimensional reduction gives rise to μ_i 's proportional to gravitino mass, we need to assume an origin of non-vanishing $\langle W^* \rangle$. It will depend on the assumption whether there are tree-level contributions or not. In this article, we study only one possibility for the origin of $\langle W^* \rangle$, just to illustrate what one should consider to guess the tree-level contributions to μ_i 's.

Let us suppose that $\langle W^* \rangle$ originates from a 3-form flux $\int_X \overline{\Omega} \wedge H \neq 0$ [26]. If a mass term of left-handed chiral fermions is to come through dimensional reduction, then there should be two gauge fermions $\Psi_{\bar{\alpha}} \equiv \sum_{a=1,2,3} \Psi^a e_{\bar{\alpha}a}$ in a term of effective action on 9+1 dimensions. Since the mass parameter is supposed to be proportional to gravitino mass, a 3-form vev $\langle H_{\alpha\beta\gamma} \rangle$ should also be involved. In the 4+1 model, we further need a vev of an anti-chiral multiplet $\overline{N}^{c\dagger}$, which originates from a vector field A_α vev. In order to write down such a term that is at least invariant under the holonomy $\text{SU}(3)$, it must contain at least three derivatives; candidates of such terms that appear first in α' -expansion are of the form

$$\frac{1}{\alpha' g_s^2} \text{tr} (D_\alpha \Psi_{\bar{\alpha}} D_\beta \Psi_{\bar{\beta}} D_{\bar{\gamma}} A_\gamma) H_{\delta\epsilon\zeta} d^{10}y, \quad (2.88)$$

with $\text{SU}(3)$ indices contracted by metric $h_{\alpha\bar{\alpha}}$, holomorphic 3-form $\Omega_{\alpha\beta\gamma}$ and its Hermitian conjugate $\overline{\Omega}_{\bar{\alpha}\bar{\beta}\bar{\gamma}}$. This term is proportional to $1/g_s^2$ because it is supposed to be in the tree-level effective action. A dimensionful coefficient $1/\alpha'$ was supplied based on dimensional analysis.

One can see, however, that such terms cannot be made $\text{SO}(6) \simeq \text{SU}(4)$ invariant. To see this, note first that the $\text{SU}(3)$ indices in (2.88) can be contracted by one $\overline{\Omega}_{\bar{\delta}\bar{\epsilon}\bar{\zeta}}$ and some metrics $h_{\alpha\bar{\alpha}}$. Since $\text{SU}(4)$ contains $\text{SU}(3) \times \text{U}(1)$, (2.88) should be neutral under the $\text{U}(1)$, if it is to be made invariant under $\text{SO}(6) \sim \text{SU}(4)$. Remembering how spinor and vector objects transform³² under $\text{SU}(3) \times \text{U}(1)$, one finds that (2.88) with an $\overline{\Omega}_{\bar{\delta}\bar{\epsilon}\bar{\zeta}}$ has +6 units of the $\text{U}(1)$ charge. Thus, (2.88) cannot be made invariant under $\text{SU}(3) \times \text{U}(1) \subset \text{SU}(4)$, no matter how the indices are

³² A spinor field $\Psi_{\bar{\alpha}}$ is in the $\mathbf{\bar{3}}^1$ representation of $\text{SU}(3) \times \text{U}(1)$, vector-type objects such as D_α and A_α transform as $\mathbf{3}^{+2}$, and anti-vector-type objects such as $D_{\bar{\alpha}}$ as $\mathbf{\bar{3}}^{-2}$.

contracted by metric. Needless to say, there is no term of the form (2.88) that is invariant under $\text{SO}(9, 1)$.

There is no tree-level contribution even from higher order terms in $\mathcal{O}(\alpha' D^2)$ expansion. This is because $D_\alpha D_{\bar{\alpha}}$ is neutral under $\text{U}(1)$, and so is $(D_{\bar{\alpha}})^3$ accompanied by $\Omega_{\alpha\beta\gamma}$.

In the 3+2 model, on the other hand, it is vev's of chiral multiplets \bar{N} that are inserted in (2.82). Thus, a Lorentz invariant operator can exist in 9+1 dimensions at tree level,

$$\frac{1}{\alpha'^3 g_s^2} \text{tr} \left(\bar{\Psi}_{\bar{\alpha}} \left(\Gamma^{\bar{\beta}} A_{\bar{\beta}} \right)^{\bar{\alpha}\bar{\gamma}} \Psi_{\bar{\gamma}} \right), \quad (2.89)$$

which is nothing but a part of the kinetic term of gauginos in 9+1 dimensions. This term has become a part of the superpotential (2.9). But it was a part of the assumptions of the 3+2 model that the massless doublet \hat{H}_0 remains in the low-energy spectrum because of rank reduction in the coupling $W \ni y^\nu \hat{H}_0 \bar{N}_0 \bar{\mathbf{5}}_0$. Therefore, the existence of (2.89) in 9+1 dimensions is irrelevant to the question of whether the L_i – H_u mass term is generated at tree level; even if the vev of gauge field moduli \bar{N}_0 's shift by of order $m_{3/2}$ in the presence of SUSY-breaking, nothing happens because the coupling is absent. Higher order terms in α' -expansion may give rise to the tree-level $\hat{\mathbf{5}}_0$ – \hat{H}_0 mass term in the 3+2 model as well;

$$\frac{1}{\alpha' g_s^2} \text{tr} \left(D_\alpha \bar{\Psi}_{\bar{\alpha}} D_{\bar{\beta}} A_{\bar{\beta}} D_\gamma \Psi_{\bar{\gamma}} \right) H_{\delta\epsilon\zeta} \quad (2.90)$$

can be made invariant under the holonomy $\text{SU}(3)$ after the indices are contracted by $h_{\alpha\bar{\alpha}}$, $\Omega_{\alpha\beta\gamma}$ and $\bar{\Omega}_{\bar{\alpha}\bar{\beta}\bar{\gamma}}$. Following the same argument as in the 4+1 model, however, there is no way making (2.90) invariant under $\text{SU}(3) \times \text{U}(1) \subset \text{SU}(4) \simeq \text{SO}(6)$. There are no higher order terms in $\mathcal{O}(\alpha' D^2)$ expansion in the $\text{SO}(9,1)$ -invariant effective action whose dimensional reduction gives rise to the tree-level contribution, just like in the 4+1 model.

The other issue is the contributions to the 1-loop amplitude with infinite Kaluza–Klein particles and stringy states in the loop. The 1-loop amplitudes (2.81, 2.82) are obtained by treating Kaluza–Klein towers as if they contained one (or a finite number of) Kaluza–Klein excitation(s) in each tower. Such amplitudes are UV-finite, because a superficial degree of divergence is -2 and -4 in the 4+1 and 3+2 model, respectively. But Kaluza–Klein towers contain infinite degrees of freedom on 3+1 dimensions, and their contributions are approximated by integrating loop momentum in 10D, not in 4D. Thus, the superficial degree of divergence becomes $+4$ and $+2$, respectively. These UV divergence is tamed and made finite in string theory. The most naive guess is that the estimate (2.83) is multiplied by a factor of $(M_s/M_{\text{KK}})^4$ and $(M_s/M_{\text{KK}})^2$, respectively, expecting that the UV divergence is tamed at least by stringy states at an energy scale around M_s .

It is too naive, however, to make a guess based only on the superficial degree of divergence. Let us take 1-loop correction to a gauge coupling constant as an example. 1-loop correction to $1/g_{\text{YM}}^2$ is expected naively to be of order $(M_s/M_{\text{KK}})^6$, since the superficial degree of divergence is +6 for amplitudes with only two gauge fields in the external lines in 9+1 dimensional spacetime. The 1-loop correction, however, is known in orbifold calculations and is of order $(M_s/M_{\text{KK}})^2$ [27]. This explicit example clearly indicates that the guess based on the superficial degree of divergence is too naive.

1-loop 1PI effective action on flat 9+1 dimensional spacetime can explain why the 1-loop threshold correction is of order $(M_s/M_{\text{KK}})^2$, not $(M_s/M_{\text{KK}})^6$. If 1-loop 1PI effective action had a term $1/\alpha'^3 \text{tr}(|F|^2)$, then its dimensional reduction would give rise to a threshold correction of order $(R^6/\alpha'^3) = (M_s/M_{\text{KK}})^6$. But, it is known from worldsheet calculation [28, 29] that there is no such correction. Absence of an $\mathcal{O}(M_s^6)$ term in the 1-loop effective action on flat 9+1 dimensional spacetime corresponds to the absence of an $\mathcal{O}(M_s^6)$ term in the 1-loop threshold corrections to the gauge coupling constant of compactified models. On the other hand, the 1-loop 1PI effective action on flat 9+1 dimensional spacetime contains a term [30]

$$d^{10}y \frac{1}{\alpha'} \text{tr}(|F|^2) \text{tr}(|F|^2), \quad (2.91)$$

and dimensional reduction of this term explains the threshold correction of order $(M_s/M_{\text{KK}})^2$ in compactified models.

String-scale dependence of observables (such as the gauge coupling constant) in compactified models may be determined by dimensional reduction of 1PI effective action of string theory on flat 9+1 dimensions. This prescription is known to work for threshold correction of gauge couplings. Although it is desirable to check with orbifold calculations whether this prescription is correct for other observable quantities, yet it does not seem to be terribly wrong. Flat 9+1 dimensional spacetime and its compactification share spacetime structure (including an extended supersymmetry) and interactions on it at short distance. Thus, the string scale dependence may be the same for both.

Let us use this prescription for the 1-loop amplitude of the $\bar{\mathbf{5}}\text{--}H(\mathbf{5})$ mass term; a brute force world sheet calculation would only be possible for orbifold compactification, and even in orbifold, it would take some time. If we are to use known results on 1PI effective action on 9+1 dimensions instead, we have already seen in the context of tree-level contributions that the mass term of chiral fermions in question cannot be obtained through dimensional reduction. The same is true for the 1-loop 1PI effective action as well, since the result at tree-level is based only on $\text{SO}(9, 1)$ invariance. Therefore, the 1-loop amplitudes with higher Kaluza–Klein states

and stringy states in the loop are not likely to contribute to the total amplitude. On the other hand, contributions from low-lying Kaluza–Klein states are not captured by the world-sheet calculations on a flat 9+1 dimensional spacetime, since particle spectra on compactified space are quite different from those on a flat space at that scale. These IR contributions have already been estimated earlier in this subsection. Thus, the rough estimate (2.83) is valid, providing our prescription is justified.

The same prescription can be applied to the 1-loop contribution to the μ -term in the 4+1 model that we described in section 2.2. Assuming that the non-vanishing gravitino mass is from the 3-form flux, we find that no such term can be written down in effective action on 9+1 dimensional spacetime with a coefficient of positive power of $1/\alpha' = M_s^2$. Thus, the 1-loop contributions from around the Kaluza–Klein scale dominate the amplitude, and hence it is likely that the estimate following from (2.42) remains valid.

This prescription is also applied to the R-parity violating coefficients ϵ for the kinetic mixing in (2.74). This R-parity violating effect can be erased away from the kinetic terms by field redefinition such as (2.77) and (2.79), but it reappears in mass matrices and trilinear interactions that involve massive states. Certainly no R-parity violating trilinear interactions are generated among massless modes of the MSSM after the redefinition. It also turned out in section 2.3 that mixing of massless eigenstates in the external lines are irrelevant to the effective couplings of the dimension-5 proton decay operators. This additional R-parity violating mixing, in principle, contributes to R-parity violating non-renormalizable operators in the effective theory, once the massive states are integrated out. Since we will discuss R-parity violating dimension-5 operators in section 2.5, we need an order-of-magnitude estimate of the additional R-parity violating mixing from the 1-loop amplitudes. UV-finite 1-loop amplitudes in (2.75, 2.76) take account only of contributions where low-lying Kaluza–Klein modes are running in the loop. Until now, we have postponed studying whether contributions from higher Kaluza–Klein states and stringy states dominate over those from low-lying states. We will now use the prescription to make an estimate of the UV contributions.

The 1-loop kinetic mixing (2.74) can be obtained through dimensional reduction, if the 1-loop 1PI effective action contains a term of the form

$$\frac{1}{\alpha'^{2-n}} \text{tr}(F_{\mu\bar{\alpha}}(D_\gamma D_{\bar{\gamma}})^n F_\alpha^\mu F_{\beta\bar{\beta}}) d^{10}y \quad (n \geq 0), \quad (2.92)$$

where vector indices $\alpha, \beta, \gamma, \bar{\alpha}, \bar{\beta}$ and $\bar{\gamma}$ are contracted by metric. In fact, there are no terms cubic in field strengths in the 1-loop 1PI effective action [28, 29]. Although the 1PI effective action has a term quartic in field strengths, the leading $\mathcal{O}(1/\alpha')$ term is totally symmetric in

the four field strengths [30], and is factorized into two E_8 -singlets, as in (2.91). Since none of $\bar{\mathbf{5}}^\dagger N \bar{H}$ cannot be factorized into two singlets of $U(1)_{\chi, \tilde{q}_7} \subset E_8$, the 1-loop kinetic mixing amplitudes are not obtained from dimensional reduction of quartic terms. Thus, there is not even a term proportional to α'^{-1} . All other terms with a coefficient in negative power of α' do not give rise to the kinetic mixing terms through dimensional reduction. Thus, we expect that the contribution from higher Kaluza–Klein states and stringy states is not proportional to a positive power of string scale, possibly because of some cancellation due to extended supersymmetry of those particles. The R-parity violating coefficients ϵ , therefore, will not be much larger than the contributions from low-lying Kaluza–Klein states in (2.75, 2.76), and we have an estimate

$$\epsilon \sim \frac{(y^d)^2}{16\pi^2} \frac{y^\nu \langle N \rangle}{M_{\text{KK}}}; \quad (2.93)$$

see comments after (2.83) for the meaning of $\langle N \rangle$.

R-parity violating vev's deform mass matrices at tree-level as in (2.36, 2.38, 2.58, 2.60), and the mixing angles are of the order $y^\nu \langle N \rangle / M_{\text{KK}}$. The additional mixing due to the 1-loop kinetic mixing is of order ϵ . Since the estimate (2.93) contains an extra 1-loop factor, this additional mixing (2.77, 2.79) is quantitatively negligible. Since the 1-loop mixing is qualitatively the same as the tree-level mixing, and negligible quantitatively, we will not make a distinction between the two different bases of (2.77, 2.79) in the following.

2.5 R-parity Violating Dimension-5 Operators

Low-energy effective theories are described only by light degrees of freedom, with all heavy states integrated out. Numerous non-renormalizable operators are generated as heavy states are integrated out, and naturally R-parity violation will also manifest itself in the non-renormalizable operators. There are four dimension-5 operators that are $SU(5)_{\text{GUT}}$ invariant and are odd under the R parity:

$$W_{\text{eff.}} \ni \frac{1}{M_{\text{eff.}}} \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{H}(\mathbf{5})_0 + \frac{1}{M_{\text{eff.}}} \hat{H}(\mathbf{5})_0 \hat{\bar{H}}(\bar{\mathbf{5}})_0 \hat{H}(\mathbf{5})_0 \hat{\bar{\mathbf{5}}}_0 \quad (2.94)$$

in the superpotential and

$$K_{\text{eff.}} \ni \frac{1}{M_{\text{eff.}}} \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{\bar{\mathbf{5}}}_0^\dagger + \frac{1}{M_{\text{eff.}}} \hat{\mathbf{10}}_0 \hat{\bar{H}}(\bar{\mathbf{5}})_0 \hat{H}(\mathbf{5})_0^\dagger + \text{h.c.} \quad (2.95)$$

in the Kähler potential³³ [2]. We will discuss in this subsection whether these four operators are generated. For operators that are generated, the effective mass parameters $M_{\text{eff.}}$ of these

³³Although these operators are trilinear in low-energy degrees of freedom, we only mean renormalizable trilinear ones in the superpotential by trilinear R-parity violation.

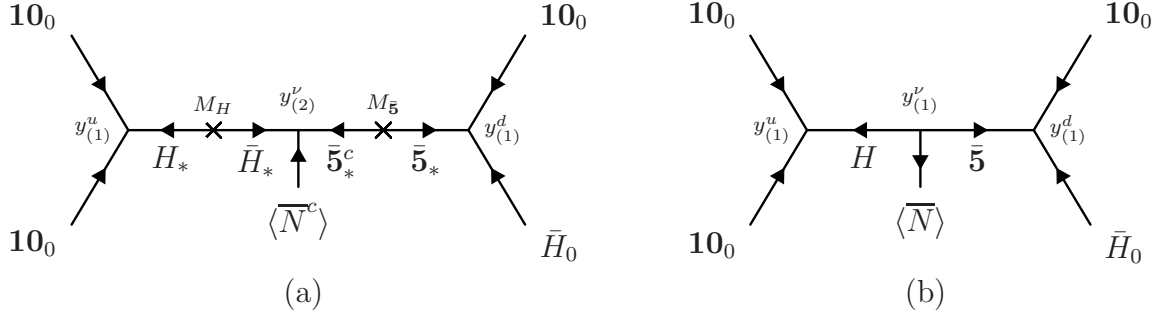


Figure 6: Typical Diagrams which generate $W \ni \mathbf{10_1 10_1 10_1 \bar{H}(5)}$ for the 4+1 model (a) and for the 3+2 model (b).

operators are expressed in terms of parameters of microscopic descriptions such as M_{KK} , $y^\nu \langle N \rangle$ etc.

2.5.1 Dimension-5 Operators in the Superpotential

Let us begin with the first operator in (2.94). This operator is generated by exchanging massive states in the $\text{SU}(5)_{\text{GUT}}-(\mathbf{5} + \bar{\mathbf{5}})$ representations, as in the super Feynman diagrams in Figure 6. Since this operator breaks R parity, R-parity violating vev's $\langle \bar{N}^c \rangle$ or $\langle \bar{N} \rangle$ have to be inserted either in the internal line or external lines in the diagram. Propagating in the internal line are massive states in the $\text{SU}(5)_{\text{GUT}}-\mathbf{5} + \bar{\mathbf{5}}$ representations. Following the argument in (2.72), one finds that diagrams with the mixing in the internal line yield

$$\frac{1}{M_{\text{eff.}}} \sim y_{(1)}^d (M^{-1})_{\bar{\mathbf{5}}H(\mathbf{5})} y_{(1)}^u \quad (2.96)$$

in both the 4+1 and 3+2 model.

In the 4+1 model, one finds that the contribution from diagrams with the mixing in the internal line becomes

$$\frac{1}{M_{\text{eff.}}} \sim y_{(1)}^d y_{(1)}^u \frac{y_{(2)}^\nu \langle \bar{N}^c \rangle}{M_{\bar{\mathbf{5}}} M_H}, \quad (2.97)$$

treating as if M_{IJ} were a 2×2 or 3×3 matrix. We have seen in section 2.3 that this finite rank treatment is sufficient in making a rough estimate of $M_{\text{eff.}}$, although there are infinite Kaluza–Klein particles. Contributions due to $\hat{\mathbf{10}}_0 - \mathbf{10}'_*$ in the external lines are of order

$$\frac{1}{M_{\text{eff.}}} \sim y_{(1)}^u y_{(2)}^d \frac{y_{(2)}^\nu \langle \bar{N}^c \rangle}{M_{\mathbf{10}'} M_H}, \quad y_{(2)}^u y_{(1)}^d \frac{y_{(2)}^\nu \langle \bar{N}^c \rangle}{M_{\mathbf{10}'} M_{\bar{\mathbf{5}}}}. \quad (2.98)$$

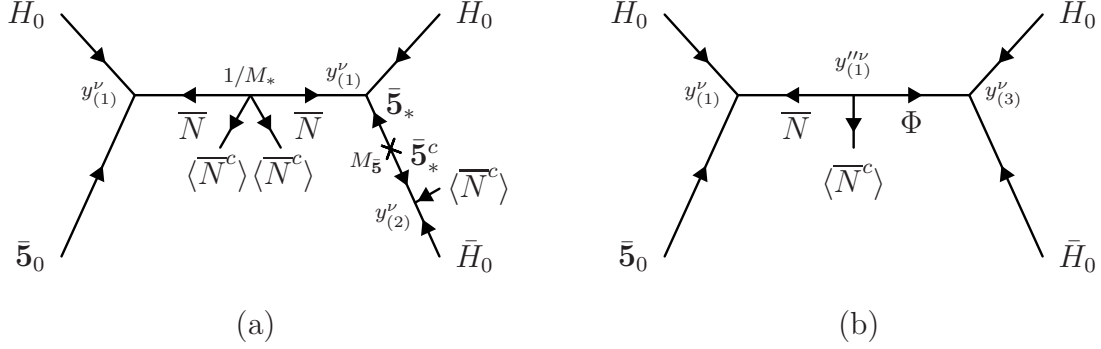


Figure 7: Examples of super Feynman diagrams which generate $W \ni \hat{H}_0(\mathbf{5}) \hat{\bar{H}}_0(\bar{\mathbf{5}}) \hat{H}_0(\mathbf{5}) \hat{\bar{5}}_0$ in the 4+1 model.

$\hat{H}(\bar{\mathbf{5}})_0 - \bar{\mathbf{5}}_*$ mixing in the external line yields an amplitude of order (2.97). If all the massive states are around the Kaluza–Klein scale, M_{KK} , then $1/M_{\text{eff.}} \sim y^u y^d (y^\nu \langle \bar{N}^c \rangle) / M_{\text{KK}}^2$. Since $1/M_{\text{eff.}} \sim y^u y^d / M_{\text{KK}}$ for the dimension-5 proton decay operators (2.63), it is likely for $y^\nu \langle \bar{N}^c \rangle \ll M_{\text{KK}}$ that the R-parity violating interaction $\Delta W = \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{\bar{H}}_0$ is weaker than the dimension-5 proton decay operators in the 4+1 model.

In the 3+2 model, note first of all that mixing in the external lines is irrelevant. Massless eigenstates $\hat{\mathbf{10}}_0$ remain pure $U(1)_{\tilde{q}_7}$ -eigenstates, and there is no mixing for these external states. The mixing of $\hat{\bar{H}}_0$ into $U(1)_{\tilde{q}_7}$ -eigenstates $\bar{\mathbf{5}}_*$ or H_*^c does not generate the operator $\Delta W = \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{\bar{H}}_0$, since we have seen in section 2.3 that there is no way to generate either $\Delta W = \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \bar{\mathbf{5}}$ or $\Delta W = \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 H^c$ in the 3+2 model. We further notice that there is no contribution from mixing in the internal line by using (2.96) and a 3×3 matrix (2.73). The same is true for the case with $a > 0$ pairs of $\bar{\mathbf{5}}^c - \bar{H}$ like states with masses of order $y^\nu \langle \bar{N} \rangle$. On the other hand, if there are $b > 0$ pairs of zero-modes $H(\mathbf{5})_0 - \bar{\mathbf{5}}_0$ that become massive in the presence of $y_{(1)}^\nu \langle \bar{N} \rangle \neq 0$, the operator $W \ni \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{\bar{H}}(\bar{\mathbf{5}})_0$ is generated, with

$$\frac{1}{M_{\text{eff.}}} \sim y_{(1)}^u y_{(1)}^d \frac{1}{y_{(1)}^\nu \langle \bar{N} \rangle}. \quad (2.99)$$

The second R-parity violating operator in (2.94) is generated through diagrams in Figure 7 in the 4+1 model and Figure 8 in the 3+2 model. There, massive $SU(5)_{\text{GUT}}$ -singlets are integrated out.

In the 4+1 model, the effective mass scale from the diagram Figure 7 (a) is approximately

$$\frac{1}{M_{\text{eff.}}} \sim (y_{(1)}^\nu)^2 \frac{y_{(2)}^\nu \langle \bar{N}^c \rangle}{M_{\bar{\mathbf{5}}}} \frac{1}{M_R} \sim \frac{(y^\nu)^2 (y^\nu \langle \bar{N}^c \rangle)}{M_{\text{KK}}} \frac{M_{\text{KK}}}{M_R}, \quad (2.100)$$

where M_R is defined in (2.35). Thus, the operator $\hat{H}_0 \hat{H}_0 \hat{H}_0 \hat{\mathbf{5}}_0$ is larger or smaller than the other R-parity violating dimension-5 operator $\hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{H}_0$ in the 4+1 model, depending on whether $M_R < M_{\text{KK}}$ or not,³⁴ if difference among various trilinear couplings and mass spectra are ignored. If there are Kaluza–Klein zero modes in \overline{N} (as assumed³⁵ in Table 2), a diagram Figure 7 (b) also contributes and

$$\frac{1}{M_{\text{eff.}}} \sim y_{(1)}^\nu y_{(3)}^\nu \frac{1}{y^{\nu\nu} \langle \overline{N}^c \rangle}, \quad (2.101)$$

which tends to be larger than (2.100); see (2.35). Therefore, in a crude approximation that all the Kaluza–Klein towers begin at a common scale M_{KK} and that $y^\nu \langle \overline{N}^c \rangle$ is somewhat smaller than M_{KK} , R-parity violating $\Delta W = \hat{H}_0 \hat{H}_0 \hat{H}_0 \hat{\mathbf{5}}_0$ is enhanced by $((y^\nu \langle \overline{N} \rangle)/M_{\text{KK}})^{-1}$ relatively to the dimension-5 proton decay operators, and the other R-parity violating operator $\Delta W = \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{H}_0$ suppressed by $((y^\nu \langle \overline{N} \rangle)/M_{\text{KK}})^{-1}$ in the 4+1 model. Here, we assume that there are zero modes of \overline{N} , and difference among all the trilinear couplings are ignored.

In the 3+2 model, an amplitude corresponding to the super Feynman diagram in Figure 8 is

$$\frac{1}{M_{\text{eff.}}} \sim (y^\nu)^2 \frac{y^\nu \langle \overline{N} \rangle}{M_{\mathbf{\bar{5}}}} \frac{1}{M_{R,\text{eff.}}}, \quad (2.102)$$

where $M_{R,\text{eff.}}$ is defined in (2.57). Thus, we see that an R-parity violating dimension-5 operator is generated in the 3+2 model, independent of high-energy spectrum in the $\text{SU}(5)_{\text{GUT}}\text{-}\mathbf{\bar{5}} + \mathbf{5}$ sector [i.e., whether $b = 0$ or not]. For cases with $b > 0$, the second term in (2.94) has larger coefficient if $M_{R,\text{eff.}} < (y^\nu \langle \overline{N} \rangle)^2/M_{\text{KK}}$, and otherwise, the first one has a larger coupling (2.99).

2.5.2 Dimension-5 Operators in the Kähler Potential

Two R-parity violating dimension-5 operators in the Kähler potential (2.95) are also generated through 1-loop diagrams in Figure 9 (in the 4+1 model) and those in Figure 10 (in the 3+2 model). Only one Feynman diagram is shown for each operator in each model, although there are many others. We roughly estimate contribution from a given diagram, by treating Kaluza–Klein towers as if they were finite number of massive states in the 3+1 dimensions. Estimates

³⁴If the low-energy neutrino masses come from the see-saw mechanism involving right-handed neutrinos, then M_R cannot be larger than 10^{15} GeV . If M_{KK} is around $M_{\text{GUT}} \sim 10^{16} \text{ GeV}$ or even larger, then $M_R < M_{\text{KK}}$.

³⁵They are necessary if masses of low-energy neutrinos are to be generated from the seesaw mechanism involving right-handed neutrinos. Majorana masses from (2.34) are not sufficient without a zero mode of \overline{N} [31].

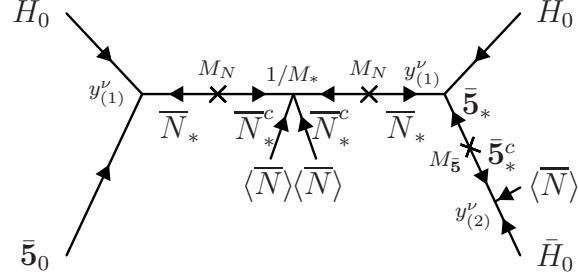


Figure 8: An example of super Feynman diagrams which generate $W \ni \hat{H}_0(\mathbf{5})\hat{\bar{H}}_0(\bar{\mathbf{5}})\hat{H}_0(\mathbf{5})\hat{\bar{5}}_0$ in the 3+2 model.

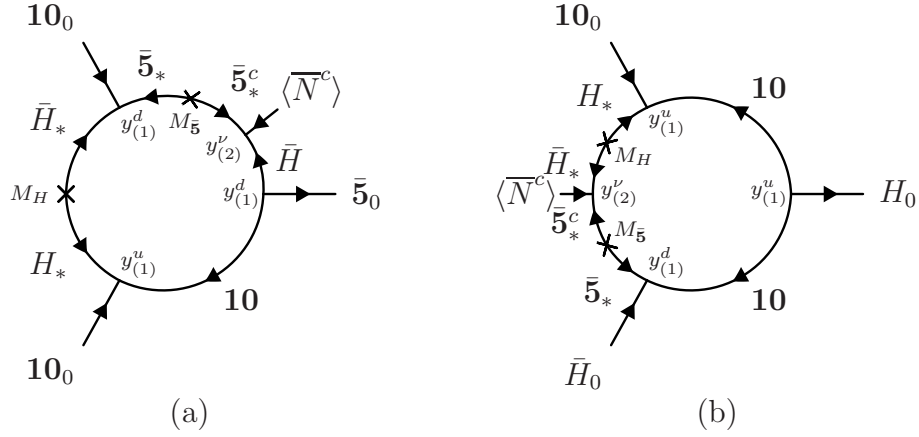


Figure 9: One-loop diagrams in the 4+1 model that generate R-parity violating dimension-5 operators in the Kähler potential.

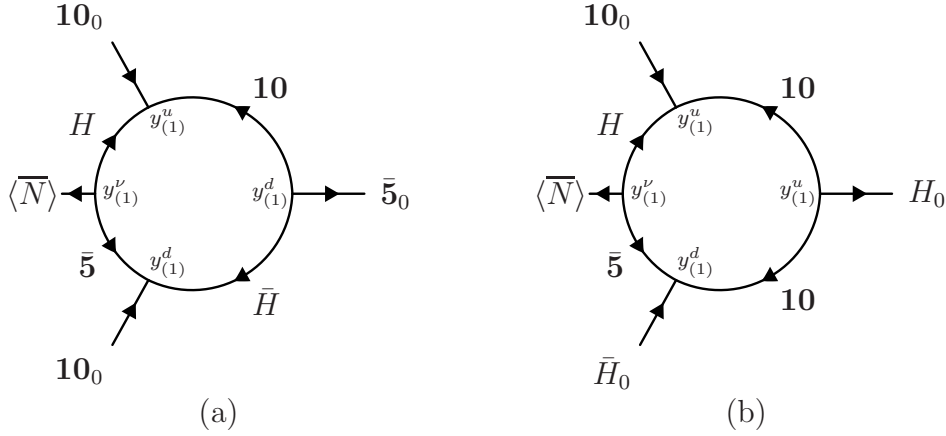


Figure 10: One-loop diagrams which generate $\mathbf{10}\mathbf{10}\bar{\mathbf{5}}^\dagger$ (a) and $\mathbf{10}\bar{H}(\bar{\mathbf{5}})H(\mathbf{5})^\dagger$ (b) for the 3+2 model.

of the UV contributions are postponed until the end of this subsection. This is just like what we did in sections 2.2 and 2.4 for other 1-loop amplitudes. To take the operator $K \ni \hat{\mathbf{10}}_0 \hat{\mathbf{10}}_0 \hat{\bar{\mathbf{5}}}_0^\dagger$ as an example, contributions from diagrams in Figure 9 (a) and Figure 10 (a) are roughly of order

$$\text{4+1 model} \quad \frac{1}{M_{\text{eff.}}} \sim \frac{|y^d|^2 y^u}{16\pi^2} \frac{M_{\bar{\mathbf{5}}}^* M_H^* (y^\nu \langle \bar{N}^c \rangle)}{|M|^4}, \quad (2.103)$$

$$\text{3+2 model} \quad \frac{1}{M_{\text{eff.}}} \sim \frac{|y^d|^2 y^u}{16\pi^2} \frac{(y^\nu \langle \bar{N} \rangle)^*}{|M|^2}, \quad (2.104)$$

where $|M|$'s in the denominators are energy scales where dominant contributions come from in integrations of loop momenta in field theory on 3+1 dimensional spacetime. All the contributions, including those above, are roughly of order $1/M_{\text{KK}}$ with a 1-loop factor $y^3/(16\pi^2)$ and an extra suppression factor $(y^\nu \langle N \rangle)/M_{\text{KK}}$. Here, we assume that all the mass parameters appearing in the loop amplitudes are around a common Kaluza–Klein scale M_{KK} . We do not go into detailed discussion of whether those operators have enhanced contributions when states with masses of order $\mathcal{O}(y^\nu \langle N \rangle)$ exist.

The rough estimates above only account for 1-loop contributions from low-lying Kaluza–Klein particles. As we discussed in section 2.4, however, such contributions may not dominate in $1/M_{\text{eff.}}$ in principle. The same operators in effective theories may also have tree-level contributions. It is also possible that the 1-loop amplitudes are dominated by contributions from higher Kaluza–Klein modes and stringy modes.

First, it is easy to see that super Yang–Mills theory on 9+1 dimensions does not give rise

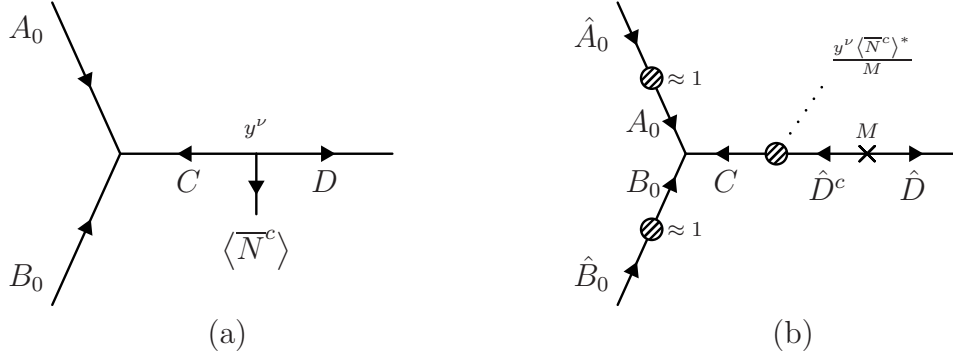


Figure 11: Starting with a trilinear superpotential $W \ni A_0 B_0 C + y^\nu C D_0 \langle N \rangle$, one might think that an effective operator $K \ni A_0 B_0 D_0^\dagger$ could be generated in the Kähler potential as in the diagram (a). However, such a trilinear term does not exist when the state going to the right is a massless eigenstate. We are saying that the amplitudes in Figure 7 and 8 with mixing in the external lines do not vanish for massless states, while the amplitude in (a) does. To understand this subtlety better, it helps to draw Feynman diagrams in terms of mass eigenstates as in (b). It is clear that a mass flip is necessary on the external line going to the right in (b), whereas it was not necessary in Figure 7 and Figure 8. This means that Kähler terms of the form $K \ni \hat{A}_0 \hat{B}_0 \hat{D}_0^\dagger$, with all of external fields being massless, cannot be generated from a diagram like (a).

to the effective operators (2.95) at tree-level; here, by super Yang–Mills theory, we only mean all the interactions that are derived from (Kaluza–Klein decomposition of) the leading order action on 9+1 dimensions, $(1/(\alpha'^3 g_s^2)) \text{tr}(|F|^2) d^{10}y$. Supergraphs like those in Figure 11 (a) exist only when one of chiral multiplets in the external lines is a massive mode. If all the external states are massless eigenmodes, such amplitudes vanish. For more detailed explanation, see the caption of Figure 11.

Tree-level contributions to the effective interactions (2.95) may also come from dimensional reduction of higher-order terms in α' -expansion in the tree-level effective action on 9+1 dimensions. Let us take

$$4 + 1 \text{ model : } \quad \bar{\mathbf{5}}^\dagger \mathbf{1010} \langle \overline{N}^c \rangle |_D = \partial_\mu \bar{\mathbf{5}}^\dagger \partial^\mu \mathbf{1010} \langle \overline{N}^c \rangle + \dots, \quad (2.105)$$

$$3 + 2 \text{ model : } \quad \bar{\mathbf{5}}^\dagger \langle \overline{N} \rangle^* \mathbf{1010} |_D = \partial_\mu \bar{\mathbf{5}}^\dagger \langle \overline{N} \rangle^* \partial^\mu \mathbf{1010} + \dots \quad (2.106)$$

in the effective theory on 3+1 dimensions as an example for concreteness; all the fields in the right-hand sides of (2.105, 2.106) are complex scalars in given representations. Such interactions can be obtained through dimensional reduction, if string theory effective action has a term of

the form

$$\frac{1}{\alpha' g_s^2} d^{10}y \operatorname{tr} (F_{\mu\alpha} F_{\bar{\alpha}}^{\mu} F_{\beta\bar{\beta}} F_{\gamma\bar{\gamma}}), \quad (2.107)$$

with indices $\alpha\text{--}\gamma$ and $\bar{\alpha}\text{--}\bar{\gamma}$ contracted covariantly by metric $h_{\alpha\bar{\alpha}}$. It is important that all the four field strength tensors are in a single trace, not factorized into two. This is because there is no way separating four representations $\mathbf{5}^{-3}$, two $\mathbf{10}^{-1}$'s and $\mathbf{1}^{+5}$ in (2.105) [resp. $\mathbf{5}^{+1}$, two $\mathbf{10}^{-3}$'s and $\mathbf{1}^{+5}$ in (2.106)] into two singlets of $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{U}(1)_{\chi} \subset E_8$ [resp. $\mathrm{SU}(5)_{\mathrm{GUT}} \times \mathrm{U}(1)_{\tilde{q}_7} \subset E_8$]. In fact, it is known that the $\mathcal{O}(1/\alpha')$ F^4 term of the sphere amplitude is factorized into two E_8 singlets in the Heterotic $E_8 \times E'_8$ string theory [32]. Thus, the effective operators (2.105, 2.106) on 3+1 dimensions are not obtained through dimensional reduction of tree-level amplitudes, at least from $\mathcal{O}(1/\alpha')$ operators. If the effective action on 9+1 dimensions contains a term $(1/g_s^2) \operatorname{tr}(F^5) d^{10}y$, which comes at the next order in α' -expansion, then its dimensional reduction may give rise to a largest possible tree-level contribution

$$\frac{1}{M_{\mathrm{eff.}}} \sim \frac{g_{\mathrm{YM}}^2 \langle N \rangle}{M_{\mathrm{KK}}^2} \left(\frac{M_{\mathrm{KK}}}{M_s} \right)^6. \quad (2.108)$$

Here, suppression due to small overlap of wavefunctions is ignored, and hence y 's and g_{YM} are much the same in this crude approximation. Rough estimates of 1-loop amplitudes (2.103, 2.104) are $1/M_{\mathrm{eff.}} \sim g_{\mathrm{YM}}^2/(16\pi^2) \times (g_{\mathrm{YM}}^2 \langle N \rangle / M_{\mathrm{KK}}^2)$ at this level of crude approximation. Thus, although tree-level contributions do not have a 1-loop factor $g_{\mathrm{YM}}^2/(16\pi^2)$, they are suppressed by $(M_{\mathrm{KK}}/M_s)^6$ or even more, and it is not very likely that they dominate over the estimates following from (2.103, 2.104).

Let us now turn to 1-loop amplitudes. Rough estimates (2.103, 2.104) take account only of contributions with low-lying Kaluza–Klein modes running in the loop. This is why the estimates based on field theory on 3+1 dimensions are UV finite; superficial degree of divergence is -4 and -2 in the 4+1 and 3+2 model, respectively. When all the Kaluza–Klein states run in the loop, however, loop momentum is effectively integrated on a 10-dimensional space, and the superficial degree of divergence becomes $+2$ and $+4$, respectively. Naive guess would be, then, to multiply $(M_s/M_{\mathrm{KK}})^2$ and $(M_s/M_{\mathrm{KK}})^4$, respectively, to the rough estimates in (2.103, 2.104), because stringy states may set in around the string scale M_s , taming the UV divergence.

We have already seen in section 2.4, however, that this guess is too naive. We introduced a prescription in section 2.4 for how to guess whether the amplitudes $1/M_{\mathrm{eff.}}$ are enhanced by a positive power of (M_s/M_{KK}) : read off $M_s = 1/\sqrt{\alpha'}$ dependence of terms in the 1-loop 1PI effective action on 9+1 dimensions whose dimensional reduction give rise to low-energy effective interactions of our interest. This prescription is based on a belief that Calabi–Yau

compactification and a flat 9+1 dimensional spacetime have a common short-distance spacetime and interactions on it, and that compactification introduces only M_{KK} -dependence, but no extra M_s -dependence.

1-loop effective action of the Heterotic $E_8 \times E'_8$ string theory on a flat 9+1 dimensional spacetime has been calculated from torus amplitudes. $\mathcal{O}(1/\alpha')$ terms quartic in gauge field strength are symmetric for four field strengths for the torus amplitude [30], and are factorized into two E_8 -singlets,

$$\frac{1}{\alpha'} \text{tr}(|F|^2) \text{tr}(|F|^2) d^{10}y. \quad (2.109)$$

Dimensional reduction of this term does not give rise to the effective interactions (2.105, 2.106) for the same reason explained for the tree-level amplitudes. All other operators that appear at higher order in the α' -expansion do not have coefficients in negative power of α' . Therefore, the 1-loop amplitudes for the effective operators (2.105, 2.106) will not have coefficients in a positive power of M_s , and in particular, the estimates (2.103, 2.104) do not need to be changed.

2.6 Brief Summary

The minimal supersymmetric standard model has four independent chiral multiplets in the $(\mathbf{2}, -1/2)$ representation of the $\text{SU}(2)_L \times \text{U}(1)_Y$ gauge group. The dimension-4 proton decay problem implies that there must be some structure in the vector space \mathcal{L} spanned by the four chiral multiplets. The most popular approach has been to introduce an unbroken discrete symmetry such as R parity. The vector space splits into two (or more) subspaces that transform differently under the discrete symmetry transformation. In the case of an unbroken \mathbb{Z}_2 symmetry, for example,

$$\mathcal{L} \simeq \mathcal{L}^+ \oplus \mathcal{L}^- \equiv \text{Span}\{H_d\} \oplus \text{Span}\{L_1, L_2, L_3\}. \quad (2.110)$$

Chiral multiplets in the two subspaces have different interactions, and hence down-type Yukawa couplings can exist, while trilinear R-parity violating couplings do not.

An alternative approach presented in this article does not need to assume that a discrete symmetry remains unbroken. In order to solve the dimension-4 proton decay problem, the vector space \mathcal{L} does not need to split. It is sufficient to assume that \mathcal{L} has a subspace

$$\mathcal{L} \supset \mathcal{L}^L; \quad (2.111)$$

chiral multiplets that belong to the subspace \mathcal{L}^L have restricted interactions. Three independent degrees of freedom in \mathcal{L}^L are identified with the lepton doublets, and the remaining one degree

of freedom in $\mathcal{L}/\mathcal{L}^L \simeq \mathcal{L}^{H_d}$ with the down-type Higgs doublet. It is not strictly necessary that \mathcal{L} has a subspace like \mathcal{L}^+ for H_d that is characterized by its restricted interactions. Suppose that a theory has a U(1) gauge symmetry with a negative [resp. positive] Fayet–Iliopoulos parameter, and there are chiral multiplets \overline{N}^c with a positive charge and \overline{N} with a negative charge. Then, \overline{N}^c [resp. \overline{N}] develops a non-vanishing expectation value, absorbing the Fayet–Iliopoulos parameter in the U(1) D-term potential. If there is an interaction (2.34) in the superpotential, then the vev of \overline{N} [resp. \overline{N}^c] is set to zero dynamically because of F-term potential coming from (2.34). The U(1) symmetry broken only by positively charged [resp. negatively charged] chiral multiplets leaves a structure (2.111) in \mathcal{L} , and similar ones in all the other vector spaces of massless modes of given representations of $SU(3)_C \times SU(2)_L \times U(1)_Y$ as well. See (2.40, 2.59, 2.61), and discussions that follow. Trilinear R-parity violating operators (1.1) are absent, no matter how large the Fayet–Iliopoulos parameter and $\langle \overline{N}^c \rangle$ [resp. $\langle \overline{N} \rangle$] are. At the same time, the interaction (2.34) gives rise to Majorana masses for right-handed neutrinos. Therefore, the absence of trilinear R-parity violating terms of massless modes is closely related to the Majorana masses.

If the Heterotic string theory is compactified with a vector bundle given by an extension, then one always has the U(1) gauge symmetry, Fayet–Iliopoulos parameter and interaction (2.34). Thus, dimension-4 proton decay problem is solved (see footnotes 14 and 21), and Majorana masses are generated for right-handed neutrinos. One does not need to choose a vacuum by hand only from special points in the moduli space, since an extra unbroken symmetry is not necessary any more.

Since the U(1) symmetry is broken spontaneously, it does not have a control over the Kähler potential of low-energy effective theory. Kinetic mixing terms between L_i and H_d can be generated, but they can be erased away by field redefinition, while keeping the structure (2.111). See discussion that follows (2.74) for more detail. Thus, the structure (2.111) and the likes for other representations remain to be a valid solution to the dimension-4 proton decay problem.

The broken U(1) symmetry does not leave a structure like (2.111) for massive fields. Mass matrices are deformed by the vev's $\langle N \rangle$ (either $\langle \overline{N}^c \rangle$ or $\langle \overline{N} \rangle$). Once massive states are integrated out to obtain low-energy effective theory, U(1)-symmetry-breaking vev's can enter in the denominator of coefficients in the low-energy effective superpotential. Since positively [resp. negatively] charged vev's in the denominator supply negative [resp. positive] U(1) charges, simple selection rule for the superpotential based on U(1)-charge counting ceased to be valid. One cannot claim that an operator with positive [resp. negative] U(1) charges is not allowed

in the effective superpotential, even when the $U(1)$ symmetry is broken by vev's of positively [resp. negatively] charged fields.

This argument does not mean that there is no rule at all in the low-energy effective superpotential. Let us suppose that we have a superpotential of a fundamental theory with a typical mass scale M . Find vev's for all the fields, expand fluctuations around the vev's, diagonalize mass matrices, and integrate out massive states to obtain low-energy effective superpotential. There is nothing special in this process. But, it is important to note that terms trilinear in massless fluctuations remain unaffected in the last step of integrating out massive states. Thus, the selection rule using $U(1)$ -charge counting can be used to constrain trilinear terms in the low-energy effective superpotential. Combined with an observation above that $U(1)$ -breaking kinetic mixing in the Kähler potential can be absorbed by redefinition of chiral multiplets while keeping the lower triangular nature of mass matrices, we see that the selection rule can be used in eliminating the dimension-4 proton decay operators (1.1). It is not that this idea has not been presented anywhere else [6, 7, 8, 9]. But we consider that this article strengthens theoretical basis for using the $U(1)$ -charge selection rule for this purpose, by clarifying the limit of the rule as well as some subtleties and logical steps that we tend to overlook.

When it comes to dimension-5 operators, first of all, there is a good chance that operators with negative [resp. positive] $U(1)$ charges are in the low-energy effective superpotential, if holomorphic insertion of positively [resp. negatively] charged vev's can make them neutral under the $U(1)$ symmetry. Such operators may be in the superpotential of a fundamental theory from the beginning, and even if this is not the case, they tend to be generated in the process of integrating out heavy particles.

Dimension-5 operators with $n > 0$ [resp. $n < 0$] $U(1)$ charges still have a chance of being generated, if some massive states in an appropriate pair of vector-like representations have mass parameters carrying $m \geq n$ [resp. $m \leq n$] $U(1)$ charges. Those operators may be generated when those states are integrated out. If a dimension-5 operator does not satisfy this condition, it is not generated in the low-energy effective superpotential. This observation was used in one of the models described in this paper in eliminating the dimension-5 proton decay operators.

Since R-parity is broken by vev's $\langle N \rangle$, it is interesting how R-parity violation appear in low-energy effective theory. Particle contents at low-energy and their charges under the broken $U(1)$ symmetry alone do not have enough power to answer to this question. This is because, for example, we never know whether dimension-5 operators with positive [resp. negative] $U(1)$ charges are generated without knowing types of particles around the mass scale M and interactions that those particles have. Since those operators tend to have couplings enhanced by some

positive power of $(M/\langle N \rangle)$, if they are generated, such operators tend to be the most important among dimension-5 operators. Thus, we need a (well-motivated) theoretical framework that specifies types of massive particles and their interactions.

We turned to a class of Heterotic string compactification that we mentioned above. Two models (called 4+1 model and 3+2 model) that belong to this class were analyzed in this article. In the 4+1 model, R-parity violating operator $\Delta W = H(\mathbf{5})_0 \bar{H}(\bar{\mathbf{5}}) H(\mathbf{5})_0 \bar{\mathbf{5}}_0$ tends to have an enhanced coupling than the dimension-5 proton decay operators. The other R-parity violating dimension-5 operator $\Delta W = \mathbf{10}_0 \mathbf{10}_0 \mathbf{10}_0 \bar{H}(\bar{\mathbf{5}})_0$ is also generated with a suppressed coefficient. Typical effective couplings of those dimension-5 operators are of order

$$\frac{1}{M_{\text{eff.}}} \sim y_{00H}^2 \frac{1}{M_{\text{KK}}}, \quad (2.112)$$

where M_{KK} is the Kaluza–Klein scale and y_{00H} 's are trilinear couplings among two light states and one massive state. Enhancement or suppression factor is roughly given by an appropriate power of $(y \langle N \rangle / M_{\text{KK}})$. In the 3+2 model, the dimension-5 proton decay operators are absent in the effective theory. On the other hand, both R-parity violating dimension-5 operators can be generated.³⁶ Thus, the dimension-5 operator in the effective superpotential (apart from $H(\mathbf{5}) \bar{\mathbf{5}} H(\mathbf{5}) \bar{\mathbf{5}}$ that we may have already seen) with the largest coupling breaks R parity in both models.

Since the theoretical framework in this section is realized in string theory, we can discuss i) R-parity violating operators generated by loop diagrams in the effective Kähler potential, as well as ii) those that originate from higher order terms suppressed by gravitational scale (or string scale). Bilinear R-parity violation $W \ni \mu_i L_i H_u$ comes from a 1-loop correction to the Kähler potential, with μ_i proportional to the SUSY breaking. We found that tree-level contribution is absent when the gravitino mass originates from a 3-form flux. Thus, μ_i 's are suppressed by 1-loop factor and $(y \langle N \rangle / M_{\text{KK}})$ relatively to gravitino mass, so that it can be very small compared with the electroweak scale.

Both dimension-5 R-parity violating operators in the effective Kähler potential (2.95) are also generated at 1-loop level. At the crudest level of approximation, coefficients of these dimension-5 operators are of order

$$\frac{1}{M_{\text{eff.}}} \approx \frac{y_{0HH}^3}{16\pi^2} \frac{1}{M_{\text{KK}}}, \quad (2.113)$$

where now y_{0HH} 's are trilinear couplings among two heavy states and one light state. Each

³⁶An exception is the operator $\Delta W = \mathbf{10}_0 \mathbf{10}_0 \mathbf{10}_0 \bar{H}(\bar{\mathbf{5}})_0$ for the case with spectrum characterized by $b = 0$.

operator is further suppressed or enhanced in an odd power of $(y \langle N \rangle)/M_{\text{KK}}$. See section 2.5 for more details.

Sections 2.1–2.5 are written as if the theoretical framework is based on the Heterotic $E_8 \times E'_8$, string theory. But absence of (or negligible) trilinear R-parity violation and presence of all other R parity violation rely only on a few ingredients: an extra U(1) symmetry, its charge assignment, non-vanishing Fayet–Iliopoulos parameter and interactions (2.20–2.27) and (2.46–2.55). The algebra of E_8 alone is sufficient in justifying all these assumptions. Therefore, M-theory dual and F-theory dual vacua that share the E_8 algebra should also share the same qualitative conclusion (c.f. [7]). However, we have also used some properties specific to the Heterotic string theory, not just algebra of E_8 . That is where we argued that tree-level contributions to (2.74, 2.80) are absent and those to (2.95) can be negligible compared with those from 1-loop, and that the 1-loop contributions are not proportional to a positive power of string scale. Thus, one has to study such questions separately for non-Heterotic vacua.

3 Phenomenology

Most of phenomenological constraints on R-parity violating couplings that have been discussed in the literature are on the trilinear couplings in the superpotential (see [1] for example). Since the theoretical framework in the previous section predicts that trilinear R-parity violation is either absent or highly suppressed, most of them, including the dimension-4 proton decay problem, are no longer a problem. R-parity violating interactions dominantly come from bilinear terms and dimension-5 operator in this framework. In the (virtual) absence of trilinear R-parity violation, constraints and predictions on nucleon decay are totally different, and some other constraints are expressed in much simpler ways because of fewer R-parity violating parameters. It is one of the purposes of this section to summarize phenomenological constraints on R-parity violating couplings in the (virtual) absence of trilinear R-parity violation. In section 3.3, we study lower bounds on bilinear and dimension-5 R-parity violating couplings, requiring that the LSP in the visible sector decay before the period of BBN. Recent developments in understanding of constraints on hypothetical particles from the BBN are reflected in the analysis. In section 3.4, we discuss nucleon decay processes induced by squark exchange diagrams combining dimension-5 and bilinear R-parity violating operators.

The theoretical framework in the previous section not only predicts which operators are generated in low-energy effective theories. It also provided order-of-magnitude estimates of those R-parity violating couplings. This means that we can analyze whether or not it can

survive various constraints. The order-of-magnitude estimates are compared with a constraint from low-energy neutrino masses in section 3.1, and with those from washout of baryon/lepton asymmetry in section 3.2.

We are by no means the first to study phenomenology of R-parity violation in the absence of trilinear R-parity violating couplings in the superpotential. Indeed, there are a plenty of literature, especially those focusing on bilinear R-parity violation. Since R-parity violation is dominated by bilinear terms at the renormalizable level in low-energy effective theory of our framework, technical results in those literature are quite useful. The appendix provides a quick summary of such results used in this section.

3.1 Neutrino Mass

Bilinear R-parity violation in (2.86) introduces neutrino–higgsino mixings, and hence the neutrino–neutralino system gives rise to an extra see-saw contribution to the neutrino masses [10]. As the neutrino masses are bounded from above by cosmological observations, there is an upper limit on bilinear R-parity violation. Using $m_\nu < 1\text{eV}$, the limit is placed on a misalignment parameter $|\epsilon'|$ [33, 1]:

$$|\epsilon'| \lesssim 3.0 \times 10^{-6} \times \frac{1}{\cos \beta} \times \left(\frac{m_\nu}{1\text{eV}} \right)^{\frac{1}{2}}. \quad (3.1)$$

$|\epsilon'|$ measures the difference between μ_i/μ_0 and v_i/v_d ; its definition is found in [33] (and also (A.9) in the appendix). The limit (3.1) also means that bilinear R-parity violation can provide the dominant part of neutrino masses in the case that (3.1) is marginally satisfied.

Since we have obtained a crude order-of-magnitude estimate of bilinear R-parity violation μ_i in section 2, it is interesting to see if it satisfies the constraint (3.1). $|\epsilon'|$ in (3.1) is roughly of order $\mathcal{O}(|\mu_i/v|)$ in the absence of alignment (see the appendix and discussion below); here, v is the electroweak scale. Using (2.83) in $\mu_i = c_i m_{3/2}$,

$$\frac{\mu_i}{v} \sim 10^{-8} \times \left(\frac{y}{10^{-2}} \right)^2 \left(\frac{y \langle N \rangle / M_{\text{KK}}}{10^{-2}} \right) \left(\frac{m_{3/2}}{100\text{GeV}} \right). \quad (3.2)$$

In gauge mediation scenario, it is very easy to satisfy the bound if μ_i is proportional to gravitino mass, but at the same time, it is very unlikely that the neutrino mass comes from bilinear R-parity violation. In the case that $m_{3/2} \sim 100\text{GeV}$, the neutrino mass bound (3.1) is also satisfied with a safe margin for a reasonable choice $y \sim 10^{-2}$ and $y^\nu \langle N \rangle / M_{\text{KK}} \sim 10^{-2}$. In this case, however, it is hard to say whether or not the neutrino–higgsino see-saw mechanism provides one of dominant contributions to the low-energy neutrino masses, because of large theoretical uncertainties associated with (3.2).

The misalignment parameter $|\epsilon'|$ (A.9) can be smaller than $\sqrt{\sum_i |\mu_i/\mu_0|^2}$ for some scenarios of SUSY breaking [34, 35]. As we see in the appendix, each misalignment parameter ϵ'_i ($i = 1, 2, 3$) can be as small as $10^{-2} \times (\mu_i/\mu_0)$ in minimal SUGRA scenario, but it is unlikely that they are even smaller than that. Thus, the neutrino mass bound on μ_i/v may be relaxed by about two orders of magnitude.

In anomaly mediation scenario, $B_i \sim m_{3/2} \times \mu_i$, so the misalignment parameter $|\epsilon'|$ can be larger than naive expectation $\mathcal{O}(|\mu_i/\mu|)$. If B_0 is somehow of order $\mathcal{O}(v^2)$ and $\mu_0 \sim \mathcal{O}(v)$, then the misalignment parameters ϵ'_i are approximately $\epsilon'_i \sim -B_i/B_0 \approx -(m_{3/2}/v) \times (\mu_i/v) \sim 10^{+3} \times (\mu_i/v)$. Thus, in this case, the limit on μ_i/v is stronger by three orders of magnitude.

3.2 Washout of Baryon/Lepton Asymmetry

Any interactions violating either baryon or lepton number symmetry could wash out the baryon/lepton asymmetry of the universe that is once generated after inflation. Thus, such couplings should not be too large. All the bilinear R-parity violating operators,

$$\Delta W = \mu_i H_u L_i \quad (3.3)$$

and

$$\Delta V_{\text{soft}} = -B_i H_u \tilde{l}_i + m_{L0i}^2 H_d^\dagger \tilde{l}_i + \text{h.c.}, \quad (3.4)$$

break lepton number symmetry. When all the dimension-5 R-parity violating operators are rewritten in terms of MSSM chiral multiplets,

$$\frac{1}{M_{\text{eff.}}} [\mathbf{101010} \bar{H}(\bar{\mathbf{5}})]_F \rightarrow \frac{1}{M_3} [QQQH_d]_F + \frac{1}{M_4} [\bar{U}Q\bar{E}H_d]_F \equiv \mathcal{O}_3 + \mathcal{O}_4, \quad (3.5)$$

$$\frac{1}{M_{\text{eff.}}} [H(\mathbf{5})\bar{\mathbf{5}}H(\mathbf{5})\bar{H}(\bar{\mathbf{5}})]_F \rightarrow \frac{1}{M_6} [H_u L H_u H_d]_F \equiv \mathcal{O}_6, \quad (3.6)$$

$$\frac{1}{M_{\text{eff.}}} [\mathbf{10105}^\dagger]_D \rightarrow \left[\frac{1}{M_7} QQ\bar{D}^\dagger + \frac{1}{M_9} \bar{U}QL^\dagger + \frac{1}{M_{10}} \bar{U}\bar{E}\bar{D}^\dagger \right]_D \equiv \mathcal{O}_7 + \mathcal{O}_9 + \mathcal{O}_{10}, \quad (3.7)$$

$$\frac{1}{M_{\text{eff.}}} [\bar{H}(\bar{\mathbf{5}})\mathbf{10}H(\mathbf{5})^\dagger]_D \rightarrow \frac{1}{M_8} [H_d \bar{E}H_u^\dagger]_D \equiv \mathcal{O}_8, \quad (3.8)$$

it is easy to see that each operator breaks either baryon or lepton number symmetry. Requiring that they do not wash out baryon/lepton asymmetry, upper bounds on these R-parity violating couplings are obtained [36].

Tiny trilinear R-parity violation (2.85, 2.87) in the 4+1 model

$$-y^d \frac{\mu_i}{M_H} [L\bar{E}L]_F - y^d \frac{\mu_i}{M_H} [LQ\bar{D}]_F - y^d \frac{\mu_i}{M_H} [\bar{D}\bar{U}\bar{D}]_F \equiv \mathcal{O}_0 + \mathcal{O}'_0 + \mathcal{O}''_0 \quad (3.9)$$

also breaks lepton and baryon number symmetry. However, the coupling constants are so small for $M_H \gtrsim M_{\text{GUT}}$ ($M_{\text{GUT}} \sim 10^{16} \text{GeV}$) that they are irrelevant to the washout of baryon/lepton asymmetry.

Bilinear R-parity violating operators are most dangerous at late time, as interaction rates scale as $\propto T$ while the temperature T is higher than the electroweak scale, whereas the Hubble parameter scales as T^2 during radiation dominance. Assuming that baryon/lepton asymmetry was generated when the temperature was higher than the electroweak scale, and requiring baryon/lepton asymmetry not to be washed out, one can obtain upper limits on bilinear R-parity violation. Applying the results of [37] to the case without trilinear R-parity violation,

$$\min_{i=1,2,3} \left(\frac{\mu_i}{\mu_0} \right) \lesssim 10^{-7} \times (y_3^d)^{-1} \sim 3 \times 10^{-6} \times \cos \beta, \quad (3.10)$$

where $y_3^d = y_b$ is the bottom Yukawa coupling. The rough estimate (3.2) of μ_i satisfies this constraint easily. Constraints on soft bilinear R-parity violating parameters in [38] are a little stronger:

$$\left(\frac{B_i}{B_0} - \frac{\mu_i}{\mu_0} \right) \lesssim 10^{-7}. \quad (3.11)$$

As long as $B_i \approx \mathcal{O}(v \times \mu_i)$, however, the estimate (3.2) also satisfies this constraint from washout.

Dimension-5 operators, on the other hand, are more relevant at higher temperature. Interaction rates of R-parity violating processes scale as $\Gamma \sim 10^{-2} T^3 / M_i^2$ ($i = 3, 4, 6, 7, 8, 9, 10$), and hence these operators would wash out baryon/lepton asymmetry right after it was generated, if they ever would. Requiring that R-parity violating processes caused by the dimension-5 operators are out of equilibrium at that time, it follows that [36]

$$M_i \gtrsim 10^{12} \text{GeV} \times \left(\frac{T_{\Delta B/\Delta L}}{10^{10} \text{GeV}} \right)^{\frac{1}{2}}, \quad (3.12)$$

where $T_{\Delta B/\Delta L}$ stands for the temperature of baryo/lepto-genesis.³⁷ Note that (3.12) should be satisfied for all seven operators (3.5–3.8). If the baryon asymmetry of the universe originates from thermal leptogenesis, we know that $T_{\Delta B/\Delta L} \gtrsim 2 \times 10^9 \text{GeV}$ [39]. In this scenario, all the effective mass scales M_i in (3.5–3.8) have to be larger than about 10^{12}GeV . We have seen in section 2 that the zeroth-order approximation of $1/M_{3,4,6}$ in the superpotential and $1/M_{7,8,9,10}$ in the Kähler potential are y^2/M_{KK} and $(y^3/16\pi^2)1/M_{\text{KK}}$, respectively. Therefore, it is quite easy

³⁷ Limits from the absence of washout of baryon/lepton asymmetry are more complicated if $T_{\Delta B/\Delta L} \gtrsim 10^{12} \text{GeV}$, because the sphaleron process is out of equilibrium when $T \gtrsim 10^{12} \text{GeV}$.

to satisfy the constraints from the washout of baryon/lepton asymmetry, if the Kaluza–Klein scale is around the GUT scale M_{GUT} .

3.3 LSP Decay

Given the order-of-magnitude estimate of bilinear R-parity violation in (3.2), it is very unlikely that the lightest supersymmetric particle in the visible sector (hereafter we call it the vLSP³⁸) has a lifetime much longer than the age of the universe. If the temperature of the universe was once sufficiently high, and thermal relic of vLSP is left after the temperature drops below the electroweak scale, then the relic vLSP has to have decayed before the period of BBN.

Constraints on R-parity violating couplings from BBN were discussed already in [10, 40, 41] and in more detail in other papers that followed. In this subsection, we reanalyze the BBN limits in the light of the latest understanding of the impact of hadronic energy injection or of stable charged hypothetical particles during the period of BBN. We ignore tiny couplings (2.85) that are present in the 4+1 model, and assume that all the renormalizable interactions violating R-parity originate from bilinear terms, as predicted by the framework in section 2. Limits on dimension-5 R-parity violating operators are also derived. The following study covers two typical possibilities: either the vLSP is a bino-like neutralino or a scalar tau (stau).

The vLSP is no longer a candidate of dark matter. But gravitino with $m_{3/2} \approx \mathcal{O}(1)\text{GeV}$ can be a good candidate of dark matter [42]. Axion and a strongly interacting stable particle with a mass of order 100 TeV (if such a particle exists) [43] can also be dark matter.

Neutralino Let us first begin with the bino-like neutralino vLSP. The thermal relic density of the neutralino is very model-dependent. Here we adopt the prediction of the so-called “bulk” region of mSUGRA parameter space:

$$m_{\tilde{\chi}^0} Y_{\tilde{\chi}^0} \sim 4 \times 10^{-10} \text{GeV} \left[\frac{m_{\tilde{\chi}^0}}{100 \text{GeV}} \right]^2. \quad (3.13)$$

Nucleons and anti-nucleons produced in jets from the decay of semi-stable particles would contribute to $p \leftrightarrow n$ conversion processes and change the fraction of ${}^4\text{He}$. Thus, any semi-stable hypothetical particle X with baryonic branching fraction B_h has to have short enough lifetime

³⁸ We coin a term vLSP because whether gravitino is lighter than vLSP is sometimes not quite important.

$\tau_X \lesssim 0.1\text{sec}$, as long as the relic density satisfies [44]³⁹

$$(mY)_X \times B_h \gtrsim 10^{-10}\text{--}10^{-9}\text{GeV}. \quad (3.14)$$

If there is enough mass difference between the vLSP neutralino and the LSP gravitino ($m_{\tilde{\chi}^0} - m_{3/2} \gg M_Z$), two-body decay $\tilde{\chi}^0 \rightarrow \psi_{3/2} + Z$ is possible at tree level, and $B_h \sim \mathcal{O}(1)$. Thus (3.14) is typically satisfied in this case, and $\tau_{\tilde{\chi}^0} \gtrsim 0.1\text{sec}$ would be in conflict with the primordial ${}^4\text{He}$ abundance. This sets a limit [45]

$$m_{3/2} \lesssim 0.8\text{MeV} \times |N_{1\tilde{Z}}| \times \left(\frac{m_{\tilde{\chi}^0}}{100\text{GeV}} \right)^{\frac{5}{2}} \times \left(\frac{\tau_{pn}}{0.1\text{sec}} \right)^{\frac{1}{2}}, \quad (3.15)$$

where $N_{1\tilde{Z}} = -\sin\theta_W N_{1\tilde{B}} + \cos\theta_W N_{1\tilde{W}^0}$ is the \tilde{Z} component of the neutralino vLSP.

If the mass difference is not large enough ($m_{\tilde{\chi}^0} - m_{3/2} \lesssim M_Z$), however, $\tilde{\chi}^0 \rightarrow \psi_{3/2} + \gamma$ is the only two-body decay at tree level, and baryons are not contained in the decay products of this dominant decay mode. Baryons are produced only in three-body decay processes, and baryonic branching fraction is of order $B_h \sim \mathcal{O}(10^{-3})$. In this case, (3.14) is not satisfied, and hence the limit from $p \leftrightarrow n$ conversion $\tau_X \lesssim 0.1\text{sec}$ does not apply. Deuteron production due to hadrodissociation of ${}^4\text{He}$ sets the most stringent limit instead; $\tau_X \lesssim 10^2\text{sec}$ is required for a hypothetical particle X if [44]

$$(mY)_X \times B_h \gtrsim 10^{-13}\text{GeV}, \quad (3.16)$$

and this condition is satisfied by the typical relic density of neutralino (3.13) even after multiplying $B_h \sim \mathcal{O}(1)$. Thus,

$$m_{3/2} \lesssim 24\text{MeV} \times |N_{1\tilde{\gamma}}| \times \left(\frac{m_{\tilde{\chi}^0}}{100\text{GeV}} \right)^{\frac{5}{2}} \times \left(\frac{\tau_{d\text{-}had}}{10^2\text{sec}} \right)^{\frac{1}{2}}, \quad (3.17)$$

where $N_{1\tilde{\gamma}} = \cos\theta_W N_{1\tilde{B}} + \sin\theta_W N_{1\tilde{W}^0}$.

If gravitino is not as light as specified above, a bino-like vLSP has to decay fast enough through R-parity violating operators before the period of BBN. Bilinear R-parity violation induces R-parity violating vertices $\hat{\nu}\text{--}\hat{\chi}^0\text{--}Z$ and $\hat{e}^\pm\text{--}\hat{\chi}^0\text{--}W^\mp$ in mass-eigenstate basis [46] (see also (A.42–A.44) in the appendix). If there is enough final state phase space for the two body

³⁹ The lower bound in (3.14) is sensitive to the estimate of the error of measurement of ${}^4\text{He}$ abundance, and here we adopt a conservative one in [48]. If the error estimate gets even larger, it brings up the lower bound in (3.14), but the change is rather slight. On the contrary, if the error estimate gets smaller, the constraint $\tau_X \lesssim 0.1\text{sec}$ applies to a semi-stable particle X with much smaller $(mY)_X \times B_h$.

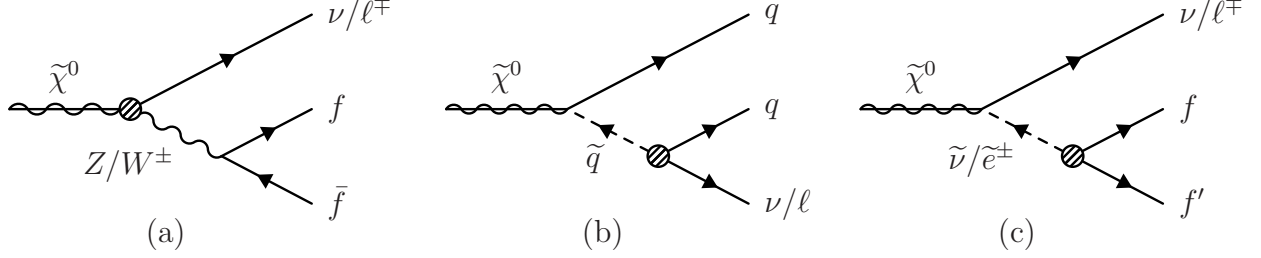


Figure 12: Feynman diagrams for three-body R-parity violating decay of neutralino. Feynman rules for the R-parity violating vertices, each shown as a blob, are found in [46, 47] and also in the appendix of this article. (a) is with a virtual Z/W^\pm , (b) with a virtual squark and (c) with a virtual non-colored scalar. (c) is only an example among several kinds of similar diagrams with a linear combination of $\phi^0 = (h^0, H^0, A^0, \tilde{\nu}_L)$, $\phi^+ = (H^+, \tilde{e}_k^c, \tilde{e}_{Lk}^*)$ or its complex conjugates ϕ^- as the virtual particle.

decay processes $\tilde{\chi}^0 \rightarrow \nu + Z$ and $\tilde{\chi}^0 \rightarrow \ell^\pm + W^\mp$ ($\ell = e, \mu, \tau$),

$$\Gamma(\tilde{\chi}_1^0 \rightarrow Z + \nu/\bar{\nu}) \sim \frac{2}{16\pi} \sum_i \left| \frac{m_{\tilde{\chi}^0}}{M_Z} \frac{g_Z}{2} (\xi_{\tilde{\nu}_i \tilde{B}} N_{1\tilde{B}})^* \right|^2 m_{\tilde{\chi}^0}, \quad (3.18)$$

$$\Gamma(\tilde{\chi}_1^0 \rightarrow W^\pm + \ell^\mp) \sim \frac{2}{16\pi} \sum_i \left| \frac{m_{\tilde{\chi}^0}}{M_W} \frac{g}{\sqrt{2}} (\xi_{\tilde{\nu}_i \tilde{B}} N_{1\tilde{B}})^* \right|^2 m_{\tilde{\chi}^0}, \quad (3.19)$$

where we have assumed a little hierarchy $M_{Z,W} \ll M_{\text{SUSY}}$ ($M_{\text{SUSY}} \approx M_{1,2}, \mu_0$); for a bino-like neutralino vLSP, the $\xi_{\tilde{\nu}_i \tilde{B}} N_{1\tilde{B}}$ terms in (A.42–A.44) are proportional to (M_Z/M_{SUSY}) , while all other terms in the vertices (A.42, A.44) are⁴⁰ to $(M_Z/M_{\text{SUSY}})^3$. Using the approximate form of $\xi_{\tilde{\nu}_i \tilde{B}}$ in (A.15) in the appendix and assuming $N_{1\tilde{B}} \sim 1$, we find that

$$\left(\frac{|\epsilon'|}{10^{-10.5}} \right) \gtrsim \left(\frac{200 \text{ GeV}}{M_1} \right)^{\frac{1}{2}} \times \frac{1}{10 \cos \beta} \times \left(\frac{0.1 \text{ sec}}{\tau_{pn}} \right)^{\frac{1}{2}}. \quad (3.20)$$

Here we used the constraint from $p \leftrightarrow n$ conversion, because $B_h \sim \mathcal{O}(1)$ in these processes, and (3.13) satisfies (3.14).

If there is not enough final state phase space for $\tilde{\chi}^0 \rightarrow \nu + Z, \bar{\nu} + Z$ and $\ell^\pm + W^\mp$, there is no two-body decay processes at tree level. Examples of Feynman diagrams for three-body decay processes are shown in Figure 12; it is easy to see that the baryonic branching fraction is of order

⁴⁰ Note that there is a partial cancellation between $U_{\tilde{e}_L i \tilde{H}_d^-} N_{1\tilde{H}_d^0}^*$ and $(\xi_{\tilde{\nu}_i \tilde{H}_d^0} N_{1\tilde{H}_d^0})^*$ in (A.44); see (A.17) and (A.22). The neutralino- W -charged-lepton vertex (A.43) is negligible compared with (A.44), because all the coefficients in (A.43) are suppressed further by m_i/μ_0 , where m_i are charged lepton masses; see (A.23).

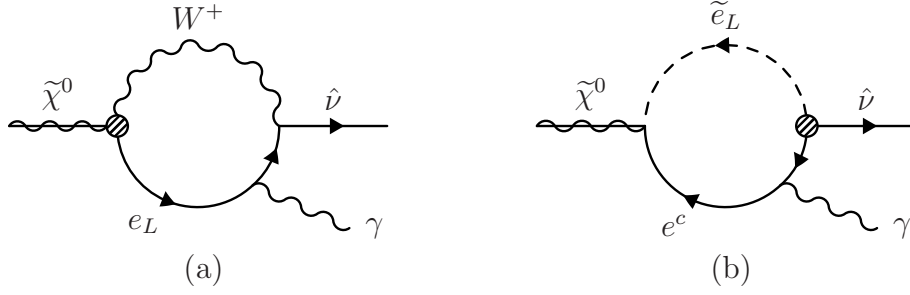


Figure 13: Feynman diagrams for R-parity violating radiative decay of neutralino. (a) is an example of diagrams with W^+ and -1 -charged fermions $\psi^- = (\widetilde{W}^-, \widetilde{H}_d^-, e_{Lk})^T$ in the loop. (b) is an example of those with charged scalar fields ϕ^\pm and charged fermions ψ^\mp , where $\psi^+ = (\widetilde{W}^+, \widetilde{H}_u^0, e_k^c)^T$.

unity, no matter which one of the diagrams (a)–(c) gives the dominant contribution. Although a kinematically allowed two-body decay process $\widetilde{\chi}^0 \rightarrow \nu + \gamma$ does not contain a baryon or even a hadron in the decay products, it is generated only at one-loop (Figure 13), and is not expected to be much faster than the three-body decay processes. Thus, the baryonic branching fraction of R-parity violating decay of neutralino vLSP is of order unity even when $m_{\widetilde{\chi}_1^0} \lesssim m_{Z,W}$. This is in contrast to the case of neutralino decay to *gravitino*. In order not to spoil the prediction of the standard BBN, we need⁴¹ $\tau_{\widetilde{\chi}^0} \lesssim 0.1$ sec, just like in the case of $m_{\widetilde{\chi}_1^0} \gg M_{Z,W}$. It is likely that the diagram Figure 12 (a) gives larger contribution to the decay rate than the others,⁴² and

$$\Gamma(\widetilde{\chi}^0 \rightarrow Z^* \nu [\bar{\nu}] \rightarrow f \bar{f} \nu [\bar{\nu}]) \sim \frac{2 \times 3.65}{192\pi^3} \sum_i \left| \frac{G_F}{\sqrt{2}} C_i^{(Z)} \right|^2 m_{\widetilde{\chi}^0}^5, \quad (3.21)$$

$$\Gamma(\widetilde{\chi}^0 \rightarrow \ell^\mp W^{\pm*} \rightarrow \ell^\mp f \bar{f}) \sim \frac{2 \times 9}{192\pi^3} \sum_i \left| \frac{G_F}{\sqrt{2}} C_i^{(W)} \right|^2 m_{\widetilde{\chi}^0}^5, \quad (3.22)$$

where $G_F/\sqrt{2} = g_Z^2/(8M_Z^2) = g^2/(8M_W^2)$. Interference between final state neutrinos in (3.21) is ignored. $C_i^{(Z)}$ and $C_i^{(W)}$ are dimensionless coefficients of $\hat{\nu}_i - \hat{\chi}_1^0 - Z$ and $\hat{e}_k^\pm - \hat{\chi}_1^0 - W$ vertices in

⁴¹ To be conservative, the constraint from $p \leftrightarrow n$ conversion should be replaced by that from excessive deuteron production through hadrodissociation, $\tau_{\widetilde{\chi}^0} \lesssim 10^2$ sec, relaxing the lower bound on the bilinear R parity violation by one order of magnitude and a half. This is because the yield of neutralino in (3.13) can be so small for $m_{\widetilde{\chi}^0} \lesssim M_{Z,W}$ that (3.14) may not be satisfied. It should also be remembered that relic density of neutralino vLSP can be even smaller than the estimate (3.13) in some parameter region of the SUSY breaking.

⁴² The amplitude for Figure 12 (c) is proportional to a Yukawa coupling, and will remain relatively small unless $\tan\beta$ is very large.

(A.42) and (A.44), respectively, and are of the order ϵ'_i or μ_i/μ_0 . Thus, we conclude that

$$\left(\frac{\sqrt{|C^{(W)}|^2 + 0.4|C^{(Z)}|^2}}{10^{-10}} \right) \gtrsim \left(\frac{0.1 \text{sec}}{\tau_{pn}} \right)^{\frac{1}{2}} \times \left(\frac{100 \text{GeV}}{m_{\tilde{\chi}^0}} \right)^{\frac{5}{2}}. \quad (3.23)$$

if $m_{\tilde{\chi}_1^0} \lesssim M_{Z,W}$.

The bino-like neutralino vLSP can also decay through dimension-5 operators violating R-parity before the period of BBN, even if bilinear R-parity violation is too small to satisfy (3.20) or (3.23). The decay modes through $\mathcal{O}_{6,8}$ are similar to those using bilinear violation, since lepton-Higgs mixing is induced when some of Higgs multiplets are replaced by their vev's. In the case of $m_{\tilde{\chi}^0} \gg M_{W,Z}$, two-body decay processes of a neutralino vLSP $\tilde{\chi}^0 \rightarrow \nu + Z$ and $\rightarrow \ell^\pm + W^\mp$ are induced by \mathcal{O}_6 . The decay widths for these modes can be calculated by using Goldstone equivalence theorem, that is, treating the longitudinal component of gauge boson in the final state as Goldstone boson, with mass M_W or M_Z and coupling equal to that of Higgs boson. These two decay modes are comparable, and the decay width is

$$\Gamma(\tilde{\chi}^0 \rightarrow \nu + Z) \simeq \frac{m_{\tilde{\chi}^0}}{16\pi} \left| 4 \frac{v_d}{M_6} N_{1\tilde{H}_u^0} \sin \beta + 2 \frac{v_u}{M_6} N_{1\tilde{H}_d^0} \sin \beta \right|^2. \quad (3.24)$$

On the other hand, the only two-body decay process induced by \mathcal{O}_8 is $\tilde{\chi}^0 \rightarrow \ell^\pm + W^\mp$, since mixing is induced only for charged leptons. The decay width is

$$\Gamma(\tilde{\chi}^0 \rightarrow \ell^\pm + W^\mp) \simeq \frac{m_{\tilde{\chi}^0}}{16\pi} \left| \frac{m_{\tilde{\chi}^0}}{M_8} N_{1\tilde{H}_u^0} \cos \beta \right|^2. \quad (3.25)$$

Applying the constraint $\tau_{\tilde{\chi}^0} \lesssim 0.1 \text{ sec}$, we have

$$M_6 \lesssim 10^{13} \text{GeV} \times (10 \cos \beta) \left(\frac{m_{\tilde{\chi}^0}}{100 \text{GeV}} \right)^{-1/2}, \quad (3.26)$$

$$M_8 \lesssim 5 \times 10^{12} \text{GeV} \times (10 \cos \beta) \left(\frac{m_{\tilde{\chi}^0}}{100 \text{GeV}} \right)^{1/2}, \quad (3.27)$$

where we assumed that $m_{\tilde{\chi}^0} \simeq M_{\tilde{B}}$ and

$$N_{1\tilde{H}_u^0} \simeq -\frac{M_Z}{M_{\tilde{B}}} \sin \theta_W \sin \beta, \quad N_{1\tilde{H}_d^0} \simeq \frac{M_Z}{M_{\tilde{B}}} \sin \theta_W \cos \beta. \quad (3.28)$$

In the case of $m_{\tilde{\chi}^0} \lesssim M_{W,Z}$, a vLSP neutralino decays to three fermions with a virtual gauge boson in the intermediate state. \mathcal{O}_6 with two H_u 's replaced by their vev's induces a mixing in

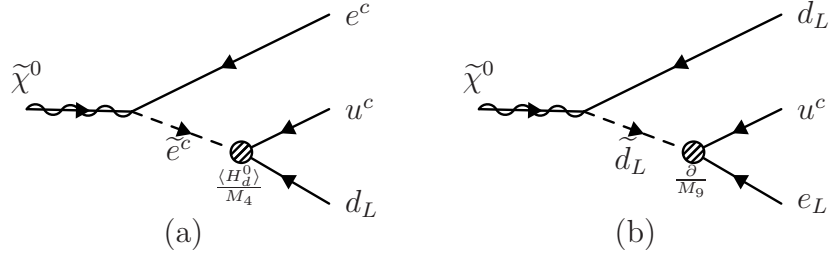


Figure 14: Examples of Feynman diagrams for neutralino decay that involve dimension-5 R-parity violating interactions. (a) uses \mathcal{O}_4 , and (b) \mathcal{O}_9 .

the neutral part $H_d^0 L_i^0$, and consequently the R-parity violating couplings $\hat{\chi}^0 - \hat{\nu} - Z$ and $\hat{\chi}^0 - \hat{\ell}^\pm - W^\mp$. We thus find that the constraint (3.23) leads to

$$M_6 \lesssim 10^{12} \text{GeV} \left(\frac{\tau_{pn}}{0.1 \text{sec}} \right)^{\frac{1}{2}} \left(\frac{m_{\tilde{\chi}^0}}{100 \text{GeV}} \right)^{\frac{5}{2}}. \quad (3.29)$$

The dimension-5 operator \mathcal{O}_8 contributes to three-body decay of vLSP neutralino only through virtual- W processes. We find, after a calculation similar to what we have had so far, that the upper bound of M_8 is

$$M_8 \lesssim 10^{11} \text{GeV} \times (10 \cos \beta) \left(\frac{200 \text{GeV}}{M_2} \right) \left(\frac{\tau_{pn}}{0.1 \text{sec}} \right)^{\frac{1}{2}} \left(\frac{m_{\tilde{\chi}^0}}{100 \text{GeV}} \right)^{\frac{5}{2}}. \quad (3.30)$$

Feynman diagrams in Figure 14 show that there are three-body decay processes for neutralino vLSP that involve dimension-5 R-parity violation $\mathcal{O}_{3,4}$ in (3.5) or $\mathcal{O}_{7,9,10}$ in (3.7). The decay width of these processes are given by (*c.f.* [49])

$$\Gamma(\tilde{\chi}^0 \rightarrow f \tilde{f}^* \rightarrow f f' f'') \sim |\lambda_{\text{eff}}|^2 |N_{1\tilde{B}}|^2 \frac{\alpha Y_f^2}{192(2\pi)^2 \cos^2 \theta_W} \frac{m_{\tilde{\chi}^0}^5}{m_0^4}, \quad (3.31)$$

where Y_f is the hypercharge of a fermion f , and m_0 the mass of a virtual sfermion \tilde{f} . Effective coupling λ_{eff} is given by $\langle H_d^0 \rangle / M_{3,4}$ for $\mathcal{O}_{3,4}$ and by $m_f / M_{7,9,10}$ for $\mathcal{O}_{7,9,10}$, where m_f is a mass of an outgoing fermion (either f' or f'' , depending on which is the heavier one). Combinatoric factors such as the number of colors and final states are ignored here because it depends on which operator is dominant and what kind of flavor structures they have. Since a pair of quark and anti-quark is always in the final states in these three-body decay processes, baryonic branching fraction is of order unity. Imposing the limit $\tau_{\tilde{\chi}^0} \lesssim 0.1 \text{ sec}$ as before, we find that

$$\lambda_{\text{eff}} > 2.3 \times 10^{-8} \frac{1}{|N_{1\tilde{B}}| Y_f} \left(\frac{100 \text{GeV}}{m_{\tilde{\chi}^0}} \right)^{\frac{5}{2}} \left(\frac{m_0}{1 \text{TeV}} \right)^2 \left(\frac{0.1 \text{sec}}{\tau_{pn}} \right)^{\frac{1}{2}}. \quad (3.32)$$

This limit corresponds to

$$M_{3,4} \lesssim 10^9 \text{GeV} \times (10 \cos \beta) \left(\frac{\tau_{pn}}{0.1 \text{sec}} \right)^{\frac{1}{2}} \left(\frac{m_{\tilde{\chi}^0}}{100 \text{GeV}} \right)^{\frac{5}{2}} \left(\frac{1 \text{TeV}}{m_0} \right)^2, \quad (3.33)$$

$$M_{7,9,10} \lesssim (10^8) \times \left(\frac{m_f}{\text{GeV}} \right) \left(\frac{\tau_{pn}}{0.1 \text{sec}} \right)^{\frac{1}{2}} \left(\frac{m_{\tilde{\chi}^0}}{100 \text{GeV}} \right)^{\frac{5}{2}} \left(\frac{1 \text{TeV}}{m_0} \right)^2 \text{GeV}. \quad (3.34)$$

Only either one of (3.33) and (3.34) has to be satisfied.

To summarize, vLSP neutralino must decay fast enough not to spoil BBN. For $m_{\tilde{\chi}^0} \gg M_Z$ [$m_{\tilde{\chi}^0} \ll M_Z$], neutralino decays to gravitino fast enough for gravitino mass satisfying (3.15) [(3.17)]. For larger gravitino mass, at least either one of R-parity violating couplings must be large enough; either one of (3.20) [(3.23)], (3.33) and (3.34) must be satisfied.

Stau Next we consider the case where the vLSP is stau. Typical thermal relic of stau is

$$m_{\tilde{\tau}} Y_{\tilde{\tau}} \simeq 7 \times 10^{-12} \text{GeV} \times \left(\frac{m_{\tilde{\tau}}}{100 \text{GeV}} \right)^2, \quad (3.35)$$

and if it decays too late, successful predictions of the BBN are no longer valid.

Problems with the BBN can be avoided for sufficiently light gravitino, as the two-body decay $\tilde{\tau} \rightarrow \tau + \psi_{3/2}$ can be fast enough for stau to decay before the period of BBN. The most stringent constraint on late-time decay $\tilde{\tau} \rightarrow \psi_{3/2} + \tau$ is to require [50, 51] that the primordial value of the abundance ratio $(n_{6\text{Li}}/n_{7\text{Li}})_p$ remains unchanged by the stau-catalyzed process in the presence of long-lived stau [52]. The upper bound is roughly $\tau_{\tilde{\tau}} \lesssim 10^3 \text{sec}$,⁴³ which corresponds to $m_{3/2} \lesssim 0.1 \text{GeV}$ for $m_{\tilde{\tau}} \sim 100 \text{GeV}$.

For gravitino mass $m_{3/2} \gtrsim 0.1 \text{GeV}$, stau must decay through R-parity violating operators instead. This requirement sets a lower bound on the R-parity violating couplings. A stau decays dominantly to a pair of leptons in the presence of bilinear R-parity violation. Although it also decays to a pair of quark and anti-quark, such an amplitude is proportional to tau lepton mass, and the branching fraction is suppressed by a factor of order $(m_{\tau} \tan \beta / m_{\text{SUSY}})^2$, which is not expected to be larger than about 10^{-2} . See [53] or the appendix for more. Since the typical yield of thermal relic of stau in (3.35) with $B_h \lesssim 10^{-2}$ does not satisfy (3.16), the deuteron constraint $\tau \lesssim 10^2 \text{sec}$ does not have to be imposed for stau vLSP decay. The most stringent limit on the stau lifetime comes from the stau-catalyzed process, and we need to require that

⁴³ The stau-catalyzed process is a problem in the range $m_{\tilde{\tau}} Y_{\tilde{\tau}} \gtrsim 10^{-13} \text{GeV}$. Typical thermal relic of stau vLSP (3.35) is within this range.

$\tau_{\tilde{\tau}} \lesssim 10^3 \text{ sec}$. Using (A.45–A.47),⁴⁴

$$\Gamma(\tilde{\tau} \rightarrow \bar{\nu}_3 \ell_k^+, \ell_3^+ \bar{\nu}_k) \simeq \frac{1}{16\pi} g_\tau^2 \left| \frac{\mu_k}{\mu_0} \right|^2 m_{\tilde{\tau}} \quad \text{for } k = 1, 2, \quad (3.36)$$

and we find that

$$\sqrt{\sum_{k=1,2} \left(\frac{\mu_k}{\mu_0} \right)^2} \gtrsim 1.8 \times 10^{-13} (10 \cos \beta) \left(\frac{100 \text{ GeV}}{m_{\tilde{\tau}}} \right) \left(\frac{10^3 \text{ sec}}{\tau_{6\text{Li}\tilde{\tau}}} \right). \quad (3.37)$$

Stau may also decay fast enough to a quark–anti-quark pair through dimension-5 operators $\mathcal{O}_{4,10}$ with

$$\Gamma(\tilde{\tau} \rightarrow q\bar{q}) \simeq \frac{3}{16\pi} \lambda_{\text{eff}}^2 m_{\tilde{\tau}}, \quad (3.38)$$

even if bilinear R-parity violation is not large enough to satisfy (3.37). The effective couplings λ_{eff} for this decay are $\lambda_{\text{eff}} \sim \langle H_d^0 \rangle / M_4$ for \mathcal{O}_4 and $\sim m_f / M_{10}$ for \mathcal{O}_{10} . If the two-body decay through \mathcal{O}_4 or \mathcal{O}_{10} is to be the solution to the BBN problem, the effective coupling of these operators has to be large enough, so that $\tau_{\tilde{\tau}} \lesssim 10^2 \text{ sec}$; since the baryonic branching fraction of the R-parity violating decay through \mathcal{O}_4 or \mathcal{O}_{10} is of order unity, (3.16) is satisfied. We thus find that

$$\lambda_{\text{eff}} \gtrsim 3.3 \times 10^{-13} \times \left(\frac{100 \text{ GeV}}{m_{\tilde{\tau}}} \right)^{\frac{1}{2}} \left(\frac{10^2 \text{ sec}}{\tau_{d\text{-}had}} \right)^{\frac{1}{2}}. \quad (3.39)$$

This lower bound on λ_{eff} is equivalent to either

$$M_4 \lesssim 5 \times 10^{13} \text{ GeV} \times (10 \cos \beta) \times \left(\frac{m_{\tilde{\tau}}}{100 \text{ GeV}} \right)^{\frac{1}{2}} \left(\frac{\tau_{d\text{-}had}}{10^2 \text{ sec}} \right)^{\frac{1}{2}}, \quad (3.40)$$

or

$$M_{10} \lesssim 3 \times 10^{12} \times m_f \times \left(\frac{m_{\tilde{\tau}}}{100 \text{ GeV}} \right)^{\frac{1}{2}} \left(\frac{\tau_{d\text{-}had}}{10^2 \text{ sec}} \right)^{\frac{1}{2}}. \quad (3.41)$$

It is sufficient if either one of these is satisfied.

The dimension-5 operator \mathcal{O}_6 induces stau decay through effective bilinear R-parity violation.⁴⁵ On the condition that $v_u \gg v_d$, dominant contribution comes from $W \ni (v_u^2 / M_6) L^0 H_d^0$,

⁴⁴ Decay amplitudes are dominated by diagrams involving the vertices (A.46), as long as we assume large $\tan \beta$. But the vertex (A.45) can be more important for small $\tan \beta$.

⁴⁵ Although \mathcal{O}_6 also provides effective trilinear R-parity violating vertices with only one of Higgs fields replaced by its vev, their contribution to stau–vLSP decay is negligible in the limit where left–right mixing is ignored, since we assume that the vLSP $\tilde{\tau}$ is the scalar partner of τ_R^c .

which causes lepton–Higgs mixing only in the neutral part. A decay mode using bino–neutrino mixing gives dominant contribution⁴⁶

$$\Gamma(\tilde{\tau} \rightarrow \tau \bar{\nu}_k) \simeq \frac{1}{16\pi} \left(\sqrt{2} g Y \right)^2 \left| \frac{m_Z}{m_{\tilde{\chi}^0}} \frac{v_u^2}{M_6 \mu_0} \sin \beta \sin \theta_W \right|^2 m_{\tilde{\tau}} \quad (3.42)$$

and the constraint from BBN is given by

$$M_6 \lesssim 4 \times 10^{13} \times \left(\frac{m_{\tilde{\tau}}}{100 \text{ GeV}} \right)^{\frac{1}{2}} \left(\frac{\mu_0}{200 \text{ GeV}} \right)^{-1} \left(\frac{m_{\tilde{\chi}}}{200 \text{ GeV}} \right)^{-1} \left(\frac{\tau_{6\text{Li}\tilde{\tau}}}{10^3 \text{ sec}} \right)^{\frac{1}{2}}. \quad (3.43)$$

The dimension-5 operator \mathcal{O}_8 also contributes to stau decay through effective bilinear R-parity violation, with one of Higgs fields replaced by its vev. In contrast to the case with \mathcal{O}_6 , lepton–Higgs mixing occurs only in the charged part. The decay width of stau vLSP depends on whether gravitino mass is large or not. In addition, if $m_{\tilde{\tau}} \gg M_Z + M_W$, a two-body decay process $\tilde{\tau}^\pm \rightarrow Z + W^\pm$ is also possible, without any of Higgs fields replaced by their vev’s.⁴⁷

To summarize, if the vLSP is stau, and if $m_{3/2} \gtrsim 100 \text{ MeV}$, one of the R-parity violating couplings should be sufficiently large, so that stau can decay fast enough and the standard predictions of the BBN are not spoiled. It is sufficient if either one of R-parity violating operators allow stau-vLSP to decay.

The neutrino mass bound (3.1) and the absence of washout of baryon/lepton asymmetry (3.10, 3.12) require that none of R-parity violating couplings are too large. On the other hand, some of R-parity violating couplings should be sufficiently large so that the vLSP can decay before the period of the BBN, unless gravitino mass is sufficiently small. Figure 17 (a)–(d) summarize the relation among these phenomenological constraints in simplified two-dimensional parameter space characterizing the order of magnitude of bilinear and dimension-5 R parity violation.

Flavor dependence of R-parity violating couplings has been ignored so far. Let the effective operator \mathcal{O}_4 in (3.5) has a flavor dependent coefficient $W \ni C_{ijk}^{(4)} Q_i \bar{U}_j \bar{E}_k H_d / M_4$, for example. It is $\sum_{i,j} |C_{ij3}^{(4)}|^2$ that determines the decay rate of stau vLSP, while what matters to the washout

⁴⁶ Another decay mode using down-type Higgsino–neutrino mixing can give comparable contribution only if $\tan \beta$ is very large.

⁴⁷ It is interesting to note that the branching fraction of various decay modes stau vLSP vary so much, depending on which R-parity violating operator is responsible primarily for the vLSP decay. \mathcal{O}_6 predicts that stau-vLSP decays dominantly into final states including τ , whereas the branching fractions of decay modes to τ and μ or e have fixed ratio if the conventional bilinear R-parity violation dominates [53]. A branching fraction to μ or e can be larger than that to τ , if \mathcal{O}_8 dominates. If R-parity violating decay of stau vLSP is observed at the LHC, branching fractions of various decay modes provide valuable information on physics behind R-parity violation.

of lepton asymmetry is the smallest among $\sum_{i,j} |C_{ijk}^{(4)}|^2$ ($k = 1, 2, 3$); baryon/lepton asymmetry is washed out completely, only when all three $B/3 - L_k$ ($k = 1, 2, 3$) symmetries are broken by some interactions in the thermal equilibrium. Therefore, with flavor structure of R-parity violation, the true allowed region in the parameter space tends to be wider than it appears in Figure 17.

3.4 Nucleon Decay

Dimension-5 R-parity violating operators \mathcal{O}_3 in (3.5) and \mathcal{O}_7 in (3.7) break baryon number symmetry, and hence proton may decay. In the 4+1 model, baryon number is broken also in the trilinear R-parity violation \mathcal{O}_0'' in (2.85). In section 3.4.1, we derive limits on R-parity violating couplings from proton lifetime. We further discuss in section 3.4.2 how to probe R-parity violation through nucleon decay experiments.

3.4.1 Limits on R-Parity Violating Couplings from Proton Lifetime

Decay products of a proton contain at least one fermion, and it must be one of e, μ, ν or their anti-particles if it is a particle in the Standard Model. Thus, proton decay is induced by squark exchange diagrams combining two R-parity violating operators; one breaks baryon number and the other lepton number. \mathcal{O}_3 , \mathcal{O}_7 and \mathcal{O}_0'' are candidates for the former, while renormalizable interactions $\tilde{u}_{L/R}^* \bar{e} P_{L/R} d$ in (A.36), $\tilde{d}_{L/R}^* \bar{e}^c P_{L/R} u$ in (A.37) and $\tilde{q}^* \bar{\nu} q + \text{h.c.}$ in (A.38–A.41) for the latter. The interactions (A.36–A.41) originate from bilinear R-parity violation. R-parity violating dimension-5 operators \mathcal{O}_4 in (3.5) and $\mathcal{O}_{9,10}$ in (3.7) can also play the same role for necessary lepton number violation⁴⁸.

Feynman diagrams in Figure 15 show squark-exchange diagrams combining baryon-number violating \mathcal{O}_3 , \mathcal{O}_7 and \mathcal{O}_0'' and a lepton-number violating (A.37). Diagrams using (A.37) contribute to nucleon decay processes such as $p \rightarrow \pi^0 + e^+$ where a positively charged lepton is in the decay products. After integrating out SUSY particles, effective operators become

⁴⁸ Dimension-5 operators $\mathcal{O}_{6,8}$ also violate R-parity and lepton number. They induce effective bilinear R-parity violating operators when some of Higgs fields are replaced with its vev. The consequence can be discussed by borrowing the constraint on bilinear R-parity violation (3.54), but it turns out that such constraint is looser if $M_{6,8}$ are assumed to be in the same order as $M_{4,9,10}$, and hence we do not discuss them here.

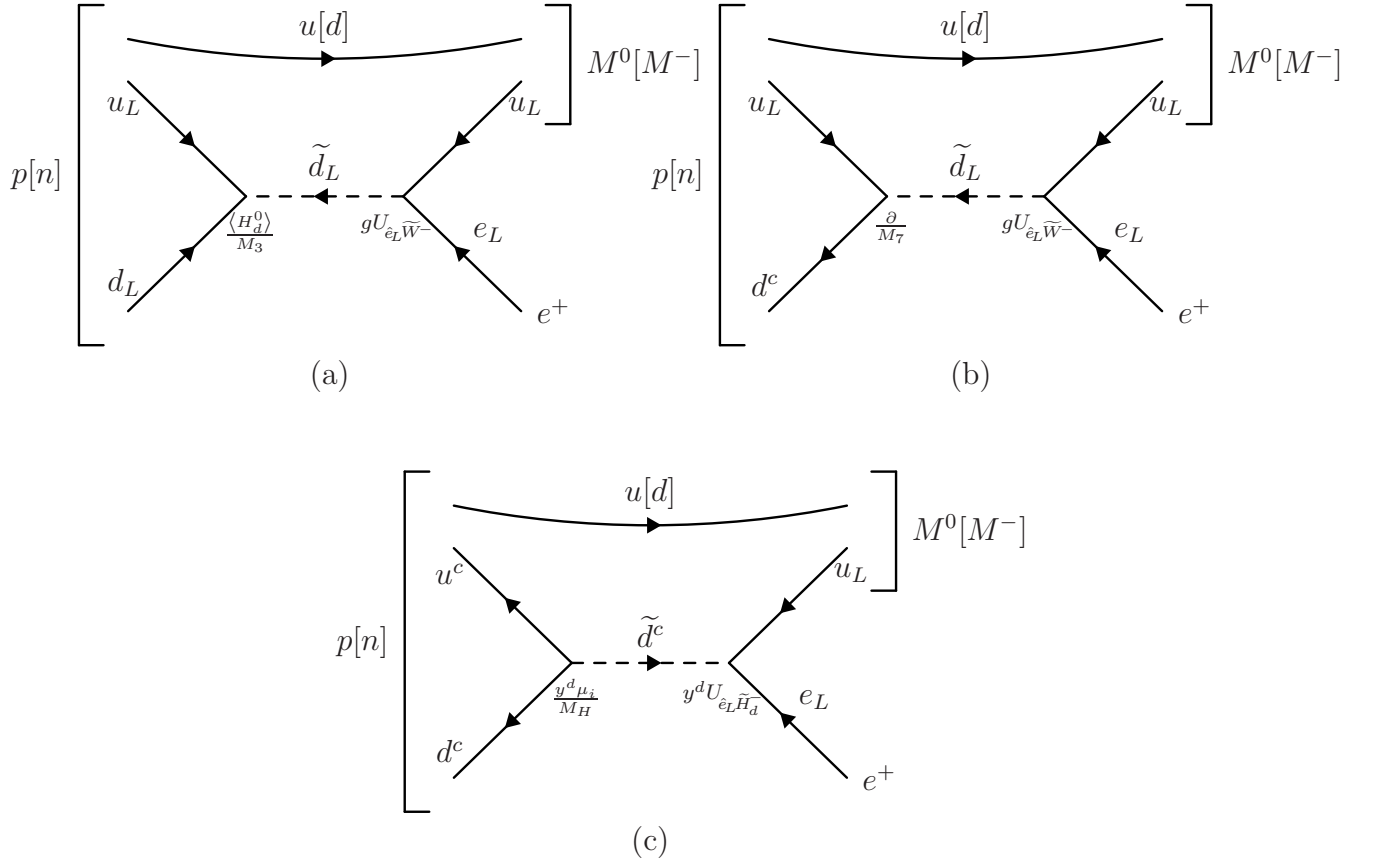


Figure 15: Feynman diagrams leading to proton decay $p \rightarrow M^0 + \ell^+$ (and neutron decay $n \rightarrow M^- + \ell^+$) that originate from the combination of two operators \mathcal{O}_3 -(A.37) in (a), \mathcal{O}_7 -(A.37) in (b) and \mathcal{O}_0'' -(A.37) in (c).

$$\mathcal{O}_3\text{--(A.37)} \quad \mathcal{L}_{\text{eff.}} = \frac{1}{m_{\tilde{d}_L}^2} \frac{v_d}{M_3} \left(gU_{\hat{e}_L \tilde{W}^-} \right) (u_L d_L)(u_L e_L), \quad (3.44)$$

$$\mathcal{O}_7\text{--(A.37)} \quad \mathcal{L}_{\text{eff.}} = \frac{1}{m_{\tilde{d}_L}^2} \frac{1}{M_7} \left(gU_{\hat{e}_L \tilde{W}^-} \right) \partial_\mu (d_R \bar{\sigma}^\mu u_L)(u_L e_L), \quad (3.45)$$

$$\mathcal{O}_0''\text{--(A.37)} \quad \mathcal{L}_{\text{eff.}} = \frac{1}{m_{\tilde{d}^c}^2} \left(-y^d \frac{\mu_i}{M_H} \right) \left(y^d U_{\hat{e}_L \tilde{H}_d^-} \right) (u_R d_R)(u_L e_L). \quad (3.46)$$

Generation indices are dropped, and the CKM matrix elements that appear in (A.37) are ignored. $d_{L/R}$ [resp. $e_{L/R}$] in this subsection just means down-type quarks [resp. charged leptons], not just the down quark [resp. electron] in the first generation. $U_{\hat{e}_L \tilde{W}^-}$ and $U_{\hat{e}_L \tilde{H}_d^-}$ are matrix elements appearing in diagonalization of charged fermion mass matrix, and are proportional to bilinear R-parity violation. For their definition, see [46, 47] or the appendix of this article.

Hadronic matrix elements

$$\langle M^0(\vec{p}) | u_L(u_L d_L) | p(\vec{k}) \rangle = W(q^2) u_L(\vec{k}), \quad (3.47)$$

$$\langle M^0(\vec{p}) | u_L(u_R d_R) | p(\vec{k}) \rangle = W'(q^2) u_L(\vec{k}) \quad (3.48)$$

have been obtained by lattice simulation [54, 55] for neutral pseudoscalars M^0 such as π^0 and K^0 , and the form factors W and W' are of order Λ_{QCD}^2 , where $\Lambda_{\text{QCD}} \sim 300 \text{ MeV}$ is the QCD scale. Although the matrix element

$$\langle M^0(\vec{p}) | u_L \partial_\mu (d_R \bar{\sigma}^\mu u_L) | p(\vec{k}) \rangle = W''(q^2) u_L(\vec{k}), \quad (3.49)$$

is not known to our knowledge, we would expect that the form factor W'' is of order Λ_{QCD}^3 .

We have discussed in the previous section how the effective mass scales of dimension-5 operators depend on parameters of microscopic descriptions such as the Kaluza–Klein scale M_{KK} and a suppression (sometimes enhancement) factor associated with a U(1) symmetry breaking ($y \langle N \rangle / M_{\text{KK}}$). Writing down the three contributions to the proton decay amplitudes in terms of those parameters, we find that they are proportional to

$$\frac{v_d}{M_3} (gU_{\hat{e}_L \tilde{W}^-}) \approx y^2 \frac{y \langle N \rangle}{M_{\text{KK}}} \frac{v_d}{M_{\text{KK}}} (gU_{\hat{e}_L \tilde{W}^-}), \quad (3.50)$$

$$\frac{\Lambda_{\text{QCD}}}{M_7} (gU_{\hat{e}_L \tilde{W}^-}) \approx \frac{y^2}{16\pi^2} y \frac{y \langle N \rangle}{M_{\text{KK}}} \frac{\Lambda_{\text{QCD}}}{M_{\text{KK}}} (gU_{\hat{e}_L \tilde{W}^-}), \quad (3.51)$$

$$y^d \frac{\mu_i}{M_H} (y^d U_{\hat{e}_L \tilde{H}_d^-}) \approx \frac{y^2}{16\pi^2} y \frac{y \langle N \rangle}{M_{\text{KK}}} \frac{m_{3/2}}{M_{\text{KK}}} (y^d U_{\hat{e}_L \tilde{H}_d^-}), \quad (3.52)$$

where we ignored a possibility that some of the dimension-5 operators are enhanced when there are vector-like pair of particles whose masses are of order $y \langle N \rangle$. Rough ratio between these

three operators is⁴⁹

$$(3.50) : (3.51) : (3.52) \\ \sim \left[(y_{00H})^2 \left(\frac{\cos \beta}{0.1} \right)^2 \right] : \left[10^{-4} \times (y_{0HH})^3 \left(\frac{\cos \beta}{0.1} \right) \right] : \left[10^{-2.5} \times (y_{00H})(y_{0HH})(y_{HHH}) \left(\frac{0.1}{\cos \beta} \right) \right], \quad (3.53)$$

where we used expressions for $U_{\hat{e}_L \tilde{W}^-}$ and $U_{\hat{e}_L \tilde{H}_d^-}$ in (A.22), and assumed that $y^d \sim m_s/v_d$ (m_s is the s quark mass). Thus the combination \mathcal{O}_3 –(A.37) is likely to contribute the most. It should be remembered, however, that the order-of-magnitude estimates in section 2 have very large uncertainties.

Experimental limits on proton lifetime are roughly around $\tau(p \rightarrow M^0 + \ell^+) \gtrsim 10^{32} - 10^{33}$ yrs. for major decay modes. This limit can be used to set an upper bound on the bilinear–dimension-5 R-parity violation. Using the partial amplitude from the combination \mathcal{O}_3 –(A.37), we find⁵⁰

$$\left(\frac{10^{15} \text{ GeV}}{M_3} \right) \left(\frac{\epsilon'_i}{10^{-9}} \right) \lesssim \frac{m_{\tilde{d}_L}^2}{(1 \text{ TeV})^2} \left(\frac{10 \text{ GeV}}{v_d} \right) \left(\frac{g\sqrt{2}M_W/M_2 \cos \beta}{10^{-1.5}} \right)^{-1}. \quad (3.54)$$

A similar constraint is obtained for the partial amplitude from \mathcal{O}_7 –(A.37) when M_3 is replaced by M_7 and v_d by Λ_{QCD} . Unless different partial amplitudes cancel one another, each of these constraints have to be satisfied, and the one with \mathcal{O}_3 gives stronger constraint if M_3 and M_7 is in the similar order.

For even smaller bilinear R-parity violation, proton decay amplitudes are dominated by squark-exchange diagrams combining two R-parity violating dimension-5 operators. Typical Feynman diagrams⁵¹ are found in Figure 16 (a)–(c). Requiring that each partial amplitude is

⁴⁹ See (2.112, 2.113) for the meaning of variations of Yukawa coupling constants y_{00H} , y_{0HH} and y_{HHH} .

⁵⁰ For consistent comparison with constraints from the vLSP decay and others, we evaluate effective mass scales like M_3 renormalized at weak scale here.

⁵¹ The 1-loop amplitude of Figure 16 (a) has a larger contribution than a tree-level one with a propagating Higgs boson replaced by its vev. The 1-loop diagram is logarithmically divergent, and the divergence is cut off at the energy scale of masses of heavy particles that are already integrated out in section 2.5. 1-loop numerical factor including logarithm and $(1/16\pi^2)$ is about 0.4, thanks to the large logarithm.

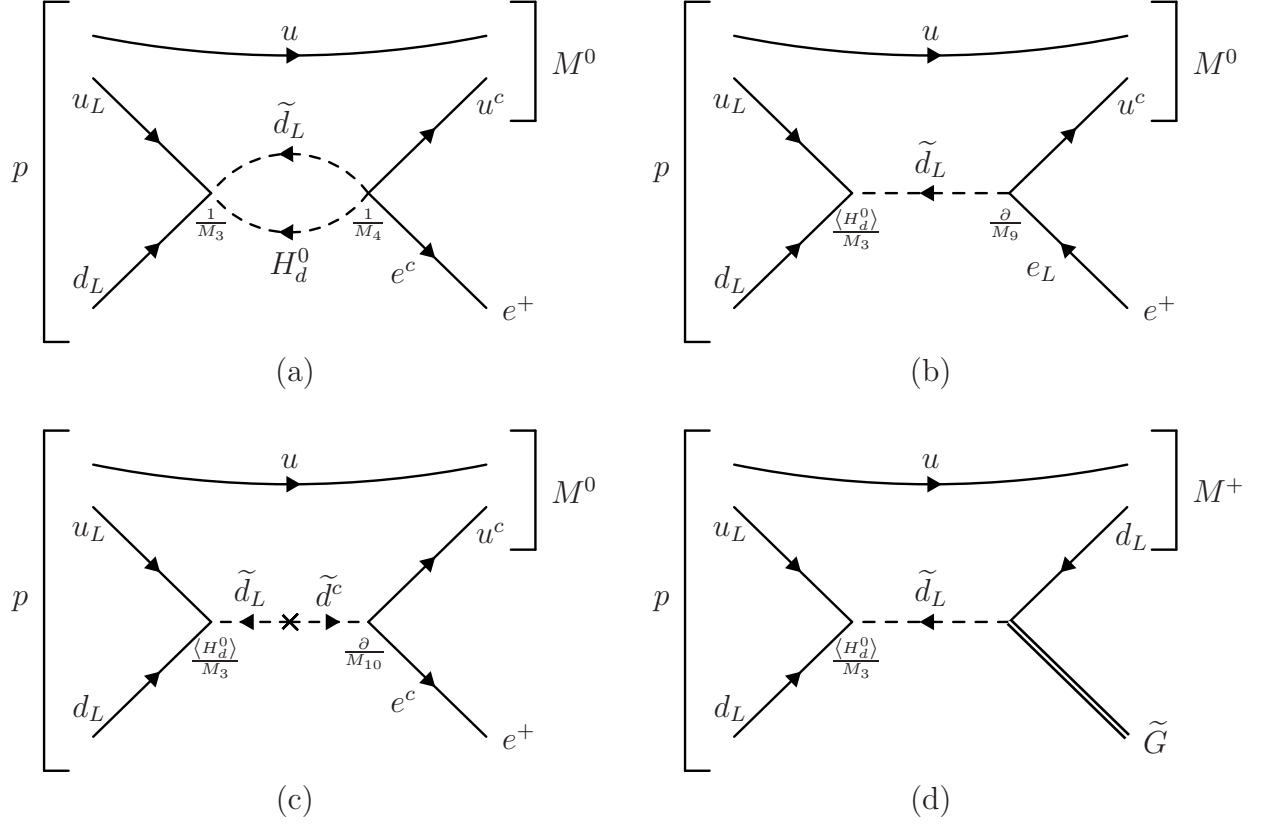


Figure 16: Feynman diagrams for proton decay involving (a): \mathcal{O}_3 – \mathcal{O}_4 , (b): \mathcal{O}_3 – \mathcal{O}_9 , (c): \mathcal{O}_3 – \mathcal{O}_{10} with left-right mixing, (d): \mathcal{O}_3 – $\mathcal{O}_{\tilde{G}}$.

small enough, we find that

$$\left(\frac{10^{15}\text{GeV}}{M_3}\right)\left(\frac{10^{15}\text{GeV}}{M_4}\right) \lesssim 1, \quad (3.55)$$

$$\left(\frac{10^{15}\text{GeV}}{M_3}\right)\left(\frac{10^{15}\text{GeV}}{M_9}\right) \lesssim \frac{m_{d_L}^2}{(1\text{TeV})^2} \left(\frac{10\text{GeV}}{v_d}\right) \left(\frac{10^{-0.5}\text{GeV}}{\Lambda_{\text{QCD}}}\right) \times 10^5, \quad (3.56)$$

$$\left(\frac{10^{15}\text{GeV}}{M_3}\right)\left(\frac{10^{15}\text{GeV}}{M_{10}}\right) \lesssim \frac{m_{\tilde{d}}^2}{(1\text{TeV})^2} \left(\frac{10\text{GeV}}{v_d}\right) \left(\frac{10^{-0.5}\text{GeV}}{\Lambda_{\text{QCD}}}\right) \frac{10^{-2}}{\theta_{LR}} \times 10^7, \quad (3.57)$$

where θ_{LR} is a mixing angle between left-handed and right-handed squarks.

If gravitino is lighter than a proton, then gravitino can be the fermion in the decay products of a proton. A squark exchange diagram in Figure 16 (d) induces proton decay, by combining baryon number violating \mathcal{O}_3 and

$$\mathcal{O}_{\tilde{G}} = \frac{i}{\sqrt{3}} \frac{1}{m_{3/2} M_G} D_\nu \tilde{q}^\dagger \psi_q \sigma^\mu \bar{\sigma}^\nu \partial_\mu \tilde{G}, \quad (3.58)$$

where \tilde{G} is the Goldstino field and (\tilde{q}, ψ_q) is a pair of complex scalar and chiral fermion of any one of chiral multiplets of the MSSM. The effective operator corresponding to the diagram is

$$\mathcal{O}_3\text{--}\mathcal{O}_{\tilde{G}} \quad \mathcal{L}_{\text{eff.}} = \frac{1}{m_{\tilde{d}_L}^2} \frac{v_d}{M_3} \frac{1}{\sqrt{3} m_{3/2} M_G} (\partial_\mu \tilde{G}_L \sigma^\nu \bar{\sigma}^\mu d_L) \partial_\nu (u_L d_L). \quad (3.59)$$

Relevant hadronic matrix element is of the form [56]

$$\langle M^+(\vec{p}) | \sigma^\nu \bar{\sigma}^\mu d_L \partial_\nu (u_L d_L) | p(\vec{k}) \rangle = p^\mu W''' u_L(\vec{k}), \quad (3.60)$$

and the form factor W''' will be of the order of Λ_{QCD}^2 . Thus, we should require that

$$\left(\frac{10^{15}\text{GeV}}{M_3}\right)\left(\frac{3\text{eV}}{m_{3/2}}\right) \lesssim \frac{m_{d_L}^2}{(1\text{TeV})^2} \left(\frac{10\text{GeV}}{v_d}\right), \quad (3.61)$$

or otherwise the partial amplitude from $\mathcal{O}_3\text{--}\mathcal{O}_{\tilde{G}}$ would predict proton decay faster than the experimental bound.

Upper bounds on dimension-5 and bilinear R-parity violation derived from proton decay are shown in Figure 17, along with constraints from neutrino mass (section 3.1), washout of baryon/lepton asymmetry (section 3.2) and BBN (section 3.3). The four parameter-space plots in Figure 17 are intended just to provide a big picture of the allowed parameter space of bilinear–dimension-5 R-parity violation. As the effective mass scale $M_{\text{eff.}}$ of dimension-5

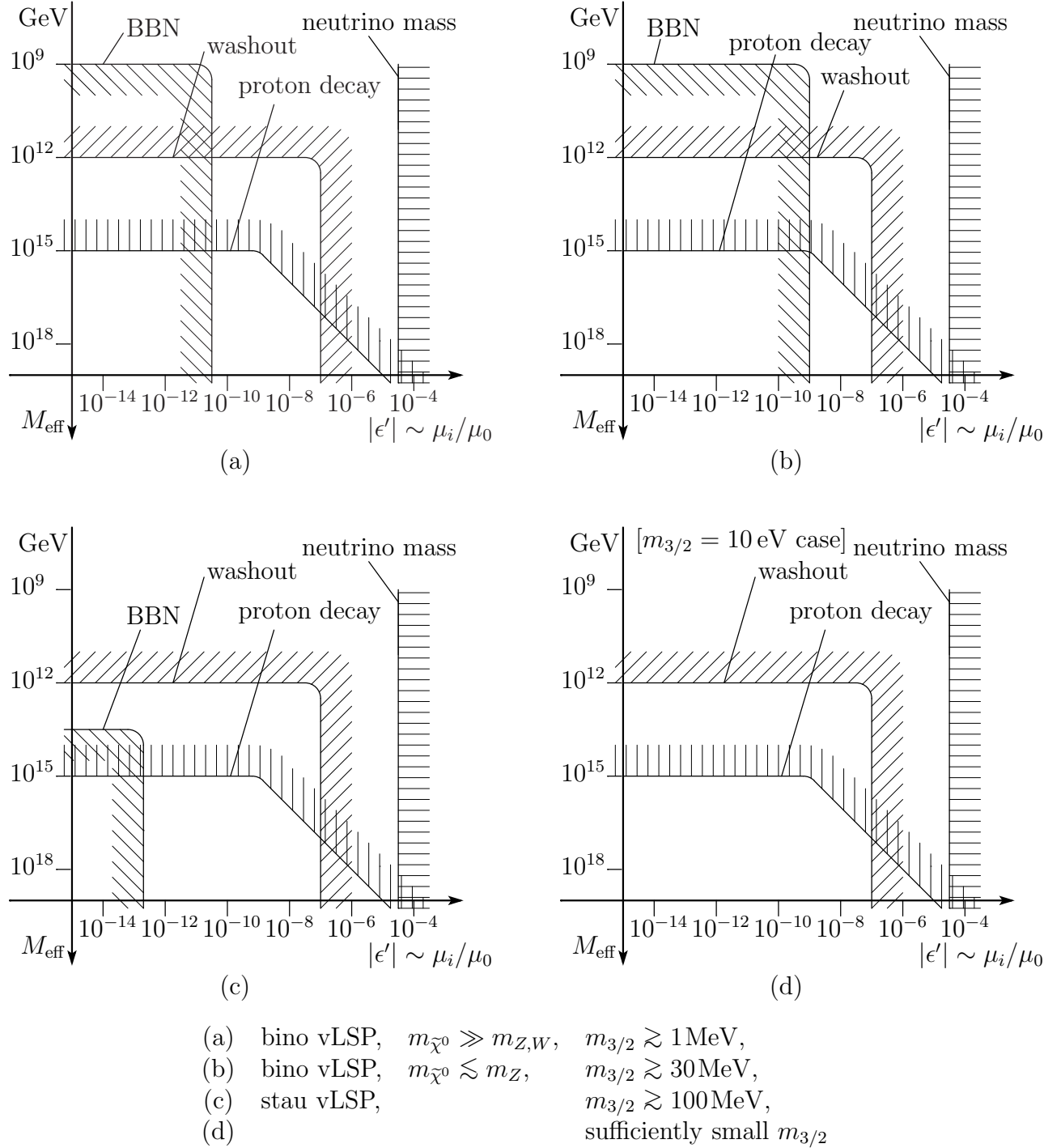


Figure 17: Simplified picture of allowed parameter space of R-parity violation. $|\epsilon'|$ parametrizes bilinear R-parity violation, and $M_{\text{eff}} \approx M_{3,4,6,7,8,9,10}$ sets the scale of dimension-5 R-parity violating operators. Hatched areas are excluded. We assumed that $T_{\Delta B/\Delta L} \sim 10^{10}$ GeV and vLSP's are thermally produced.

R-parity violation, we adopt $M_{3,4}$ of the operators $\mathcal{O}_{3,4}$ for demonstration. If $M_{3,4,6,7,8,9,10}$ are assumed to be in the same order of magnitude, the operators $\mathcal{O}_{3,4}$ are the most effective pair for proton decay. For washout constraint, there are no significant dependence on which operators are used. On the other hand, some of dimension-5 R-parity violating operators among $\mathcal{O}_{6,7,8,9,10}$ let the vLSP decay faster than through $\mathcal{O}_{3,4}$. In such cases, the constraint from BBN on dimension-5 R-parity violation is looser than shown in Figure 17, but general perspective of allowed parameter region is not largely altered.

If baryon asymmetry was generated when the temperature is below the electroweak scale, then the upper bound on the bilinear R-parity violation is replaced by the neutrino mass bound from cosmology. The allowed parameter space is extended to larger $|\epsilon'|$ by about two orders of magnitude.

Flavor structures are not taken into account in Figure 17. For the case of stau vLSP, allowed parameter space may extend to arbitrary small $|\epsilon'|$ with $M_{\text{eff.}} \approx 10^{15} \text{ GeV}$, if flavor structures of dimension-5 operators are taken into account; see the comments at the end of section 3.3.

The theoretical framework in section 2 provides rough estimates of bilinear and dimension-5 R-parity violation, $|\epsilon'| \sim \mathcal{O}(10^{-8})$ and $M_{\text{eff.}} \sim \mathcal{O}(M_{\text{KK}})$. Here $M_{\text{KK}} \sim M_{\text{GUT}}$ in a scenario where $\text{SU}(5)_{\text{GUT}}$ symmetry breaking is associated with the compactification of spacetime. It roughly corresponds to the upper-right corner of the parameter space that survives all the phenomenological constraints discussed in sections 3.1–3.4 in the big-picture parameter space in Figure 17.

3.4.2 Probing R-Parity Violation with Nucleon Decay

No evidence for proton decay has been found so far. However, once nucleon decay is discovered by experiments, then data of branching fractions of various decay modes can be used to study physics behind the nucleon decay.

Nucleon decay has been discussed in the literature mainly in the context of unified theories. GUT gauge boson exchange and colored Higgsino exchange predict dimension-6 and -5 operators in the Kähler potential and superpotential:

$$K \ni \bar{E}^\dagger Q \bar{U}^\dagger Q + \bar{D}^\dagger L \bar{U}^\dagger Q, \quad (3.62)$$

$$W \ni Q Q Q L + \bar{U} \bar{U} \bar{E} \bar{D}. \quad (3.63)$$

All these operators happen to preserve $B - L$. Therefore, $B - L$ number is preserved in all the nucleon decay processes $p[n] \rightarrow M^0[M^-] + \ell^+$ and $p[n] \rightarrow M^+[M^0] + \bar{\nu}$ which are predicted by conventional (SUSY) GUT's. It is also known [57] that the $B - L$ number is preserved in all

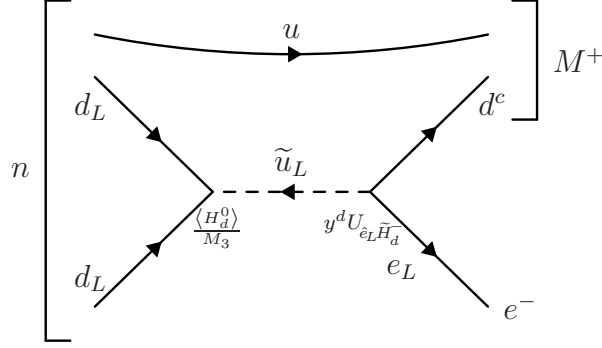


Figure 18: A Feynman diagram for $B - L$ violating neutron decay. In this example the combination of \mathcal{O}_3 –(A.36) is used.

the baryon-number violating dimension-6 operators of the Standard Model that preserve the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$.

A Feynman diagram in Figure 18 shows that a $B - L$ breaking nucleon decay $n \rightarrow M^+ + \ell^-$ is possible in the presence of R-parity violation. Squark-exchange diagrams involving $\tilde{u}^* \bar{e} d$ vertex (A.36) induce neutron decay with a negatively charged lepton in the final state. This $B - L$ breaking neutron decay, coming from a partial amplitude \mathcal{O}_3 –(A.36), is rather a robust prediction of R-parity violation. As one can see in Table 4, all but one combination of two operators for nucleon decay allow the $B - L$ breaking neutron decay in the third row.⁵² Even in gauge mediation scenario with very light gravitino of $m_{3/2} \sim \mathcal{O}(1\text{--}10\text{ eV})$, the amplitudes involving $\mathcal{O}_{\tilde{G}}$ is at most comparable with other decay modes, so $n \rightarrow M^+ + \ell^-$ may be observed with a considerable fraction.

The $B - L$ violating nucleon decay is possible because all the effective operators that we obtain after integrating out squarks are dimension-7 or higher. Either Higgs boson (vev) or derivatives are involved in those operators. For example,

$$\mathcal{O}_3\text{--(A.36)} \quad \mathcal{L}_{\text{eff.}} \sim \frac{1}{m_{\tilde{q}}^2} \frac{y^d(\mu_i/\mu_0)}{M_3} \bar{d}^c(lq)(qH_d), \quad (3.64)$$

is a dimension-7 operator in the Standard-Model fields, with the coefficient $1/m_{\tilde{q}}^2$ coming from squark exchange. After a Higgs doublet is replaced by its vev, this operator effectively becomes a 4-fermion dimension-6 operator with a coefficient suppressed only by a single power of a large

⁵²Although proton decay processes $p \rightarrow M^+ + \nu$ also break the $B - L$ symmetry, there is no way to confirm in experiments whether the missing particle is ν or something else.

	$\mathcal{O}_{3-}(\text{ren.})$	$\mathcal{O}_{7-}(\text{ren.})$	$\mathcal{O}_{3,7-}\mathcal{O}_4$	$\mathcal{O}_{3,7-}\mathcal{O}_9$	$\mathcal{O}_{3-}\mathcal{O}_{10}$	$\mathcal{O}_{7-}\mathcal{O}_{10}$	$\mathcal{O}_{3,7-}\mathcal{O}_{\tilde{G}}$
$p[n] \rightarrow M^0[M^-] + \ell^+$	✓	✓	✓	✓	(LR)	✓	
$p[n] \rightarrow M^+[M^0] + f^0$	✓	✓		✓			✓
$n \rightarrow M^+ + \ell^-$	✓	✓	(LR)	(LR)	(LR)	(LR)	

Table 4: Check marks in this table show that a combination of operators in a given column contributes to nucleon decay processes specified in the three rows. $M^{\pm,0}$ are mesons with given electric charges, $\ell^{\pm} = e^{\pm}, \mu^{\pm}$, and f^0 a neutral fermion such as $\bar{\nu}, \nu$ and \tilde{G} . “(ren.)” in the first two columns mean renormalizable interactions listed in (A.36–A.41). Some nucleon decay partial amplitudes are generated with an insertion of left-right mixing in the virtual squark propagator (see Figure 16 (c)). Such cases are indicated in the table by “(LR)”.

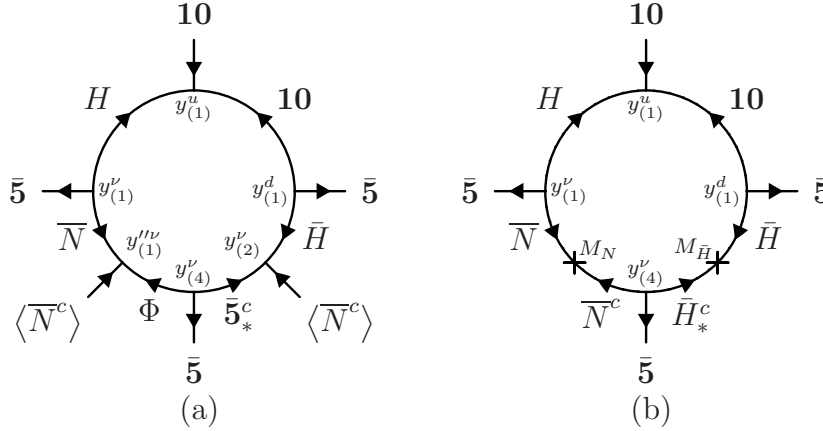


Figure 19: Feynman diagrams for $K \ni \bar{\mathbf{5}}^\dagger \bar{\mathbf{5}}^\dagger \bar{\mathbf{5}}^\dagger \mathbf{10}$, (a) for the 4+1 model and (b) for the 3+2 model.

energy scale M_3 . Thus, such an effective operator can be more important than the nucleon decay operators (3.62, 3.63) of conventional unified theories.

The $B - L$ violating neutron decay is regarded as a signal different from prediction of conventional unified theories, and certainly R-parity violation is one of possible explanations. One should keep in mind, however, that an effective dimension-6 operator

$$K \ni \bar{\mathbf{5}}^\dagger \bar{\mathbf{5}}^\dagger \bar{\mathbf{5}}^\dagger \mathbf{10} = L^\dagger \bar{D}^\dagger \bar{D}^\dagger Q + \bar{D}^\dagger \bar{D}^\dagger \bar{D}^\dagger \bar{E} + L^\dagger L^\dagger \bar{D}^\dagger \bar{U} \quad (3.65)$$

also breaks $B - L$ symmetry, while R-parity is conserved. It is true that this operator cannot be generated by gauge-boson exchange processes in any unified theories on 3+1 dimensions, as such processes would result in two anti-chiral multiplets and two chiral multiplets. But, it will be possible to come up with a model of SUSY GUT with some new matter multiplets

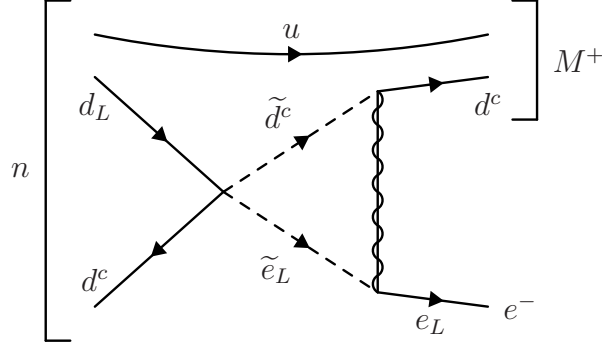


Figure 20: A Feynman diagram for $B - L$ violating neutron decay through $K \ni \bar{\mathbf{5}}^\dagger \bar{\mathbf{5}}^\dagger \bar{\mathbf{5}}^\dagger \mathbf{10}$.

and their interactions in the superpotential, so that this operator is generated.⁵³ The first two operators of the MSSM in (3.65) generate dimension-7 effective operators in the standard model, after sfermions and gauginos are integrated out (Figure 20). Therefore, just an observation of $B - L$ violating neutron decay is not enough to conclude that R-parity is not preserved. It is an interesting question whether it is possible to distinguish nucleon decay due to R-parity violation from one through (3.65), using data on branching fraction of various decay modes. But this question is beyond the scope of this paper.

It should be possible to derive predictions on flavor dependent branching fractions by exploiting flavor structure of R-parity violating operators. All three chiral multiplets Q in \mathcal{O}_3 cannot be in the same generation; one chiral multiplet Q in the second generation is necessarily involved. On the other hand, operators (A.36–A.41) and $\mathcal{O}_{\tilde{G}}$ does not introduce a generation mixing (except for predictable small flavor mixing from CKM matrix elements in (A.36, A.37)). Thus, dominant decay modes will include a K meson in the final state, if the nucleon decay is dominated by partial amplitudes \mathcal{O}_3 –(A.36–A.41) or \mathcal{O}_3 – $\mathcal{O}_{\tilde{G}}$. Quantitative predictions, however, are not covered in this article.

3.5 Brief Summary

In section 3, we studied phenomenological constraints on R-parity violation, specialized to cases when there are only bilinear and dimension-5 R-parity violation. The (virtual) absence of trilinear R-parity violation is a prediction of the theoretical framework in section 2. BBN constraints were reanalyzed in section 3.3, where we exploited the latest understanding of the

⁵³ This operator exists even in the 4+1 model and 3+2 model that we explained in section 2.2. See Figure 19.

impact of hadronic energy injection and presence of stable charged particle during the period of BBN. Section 3.4 placed a limit on bilinear–dimension-5 R-parity violation through proton decay. An allowed parameter space of bilinear–dimension-5 R parity violation is presented in Figure 17, where constraints from BBN and proton decay are shown along with those from cosmological neutrino mass bound and absence of washout of baryon/lepton asymmetry at high temperature. Theoretical framework in section 2 predicts a theoretically likely region in the parameter space (with very large uncertainties). The region is roughly around the upper right corner of the space that is still allowed by all the phenomenological constraints discussed above.

Because the likely region is around the upper right corner, the framework in the section 2 prefer stronger R-parity violation within the allowed parameter region. This means that there is a chance that R-parity violation is confirmed by experiments. R-parity violating decay of the LSP in the visible sector (vLSP) may be observed inside the detectors of the LHC for large bilinear R-parity violation. Studies of this signal of R-parity violation are found in the literature, and we have nothing to add in this article. The R-parity violating decay of vLSP may not be observed in accelerators, however, for smaller bilinear R-parity violation and/or light gravitino in the gauge mediation scenario.

As we discussed in section 3.4, nucleon decay can be an alternative way to probe R-parity violation. Especially, non-vanishing branching fraction of $B - L$ breaking neutron decay $n \rightarrow M^+ + \ell^-$ is a robust prediction of bilinear–dimension-5 R-parity violation. $n \rightarrow M^+ + \ell^-$ is always predicted except in the decay mode into gravitino, and the decay ratio into gravitino can be at most comparable with other decay mode, even if gravitino mass is very small as $\mathcal{O}(1\text{--}10\text{eV})$. This is a notable feature of bilinear–dimension-5 R-parity violation because the observation of $n \rightarrow M^+ + \ell^-$ enables us to distinguish it from conventional (SUSY) GUT's, which predict only $B - L$ preserving nucleon decay processes. Therefore, nucleon decay experiments can be complementary to the R-parity violating decay of the vLSP in the detectors of the LHC in probing R-parity violation.

Acknowledgments

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A Notes on Bilinear R-parity Violation

In the MSSM with bilinear R-parity violation, superpotential is given by

$$W = y_{ij}^u \bar{U}_i Q_j H_u - y_{kj}^d \bar{D}_k Q_j H_d - y_{kj}^e L_k \bar{E}_j H_d + \mu_0 H_u H_d + \mu_i H_u L_i, \quad (\text{A.1})$$

and soft SUSY breaking potential by

$$V_{\text{soft}} = m_{Q_{ij}}^2 \tilde{q}_i^\dagger \tilde{q}_j + m_{D_{ij}}^2 \tilde{d}_i^\dagger \tilde{d}_j + m_{U_{ij}}^2 \tilde{u}_i^\dagger \tilde{u}_j + m_{E_{ij}}^2 \tilde{e}_i^\dagger \tilde{e}_j + m_{Hu}^2 |h_u|^2 \quad (\text{A.2})$$

$$+ m_{L00}^2 |h_d|^2 + m_{L_{ij}}^2 \tilde{l}_i^\dagger \tilde{l}_j + m_{L0i}^2 h_d^\dagger \tilde{l}_i + \text{h.c.} \quad (\text{A.3})$$

$$+ B_0 h_u h_d + B_i h_u \tilde{l}_i \quad (\text{A.4})$$

$$+ A^u y^u \tilde{u}^c \tilde{q} h_u - A^d y^d \tilde{d}^c \tilde{q} h_d - A^e y^e \tilde{l}^c h_d. \quad (\text{A.5})$$

In bilinear R-parity violating scenario, there exists a basis of chiral multiplets $(H_d, L_{1,2,3})$ such that the R-parity violation appears only in bilinear terms as above. This basis is called bilinear basis. We basically follow the convention of [58].

We will assume that all the R-parity violating parameters, μ_i , B_i and m_{L0i}^2 , are small compared with μ_0 , B_0 and m_{L00}^2 and m_{Lij}^2 , respectively. Anything that are quadratic in the R-parity violating parameters are ignored in this article. Sneutrino vev's in the bilinear basis are given approximately by [34]

$$v_i \simeq \frac{B_i v_u - v_d(\mu_0 \mu_i + m_{L0i}^2)}{m_{Lii}^2 + \frac{M_Z^2}{2} \cos(2\beta)}, \quad (\text{A.6})$$

where $v_i \equiv \langle \tilde{\nu}_i \rangle = \langle L_i^0 \rangle$, $v_d \equiv \langle H_d^0 \rangle$, $v_u \equiv \langle H_u^0 \rangle$, and $\tan \beta \equiv v_u/v_d$, so $v_i \ll v_d$. We do not distinguish $\sqrt{v_d^2 + \sum_i v_i^2}$ from v_d , as the difference between them is quadratic in the small R-parity violating parameters. Using the leading-order part of the H_d^0 minimization condition

$$(m_{L00}^2 + \mu_0^2) + \frac{M_Z^2}{2} \cos(2\beta) = B_0 \tan \beta, \quad (\text{A.7})$$

the expression above can be rewritten in a useful form [35]:

$$\frac{v_i}{v_d} \simeq \frac{B_i - \cot \beta (\mu_0 \mu_i + m_{L0i}^2)}{B_0 - \cot \beta (\mu_0^2 + m_{L00}^2 - m_{Lii}^2)}. \quad (\text{A.8})$$

Misalignment parameters ϵ'_i are defined by [33]⁵⁴

$$\epsilon'_i \equiv \left(\frac{\mu_i}{\mu_0} - \frac{v_i}{v_d} \right), \quad (\text{A.9})$$

⁵⁴The misalignment parameter ξ in [33, 1] corresponds to $|\epsilon'|$, norm of ϵ'_i . Many references introduced different notations for these misalignment parameters. To name a few, Ref. [35] introduces $\alpha_i \equiv v_i/v_d$, $\gamma_i \equiv \mu_i/\mu_0$, $\delta_i \equiv B_i/B_0$, and difference between arbitrary two out of α, γ and δ is basis independent. For example, $(\gamma_i - \alpha_i) = \epsilon'_i$. Reference [46] uses $\Lambda_i = \epsilon'_i \mu_0 v_d$. ζ in [1] is norm of $(\delta_i - \alpha_i)$.

and its $\cot \beta$ expansion is given by

$$\begin{aligned} \epsilon'_i &= \left(\frac{\mu_i}{\mu_0} - \frac{B_i}{B_0} \right) \left(1 + \cot \beta \frac{\mu_0^2}{B_0} \right) - \frac{B_i}{B_0} \cot \beta \frac{m_{L00}^2 - m_{Lii}^2}{B_0} + \cot \beta \frac{m_{L0i}^2}{B_0} \\ &+ \mathcal{O} \left(\frac{\mu_i}{\mu_0} \cot^2 \beta, \frac{B_i}{B_0} \cot^2 \beta \right). \end{aligned} \quad (\text{A.10})$$

In minimal SUGRA mediation scenario, the first three terms in (A.10) vanish in the UV initial condition, and the remaining terms are suppressed by $\tan^{-2} \beta$. Thus, ϵ'_i can be much smaller than μ_i/μ_0 or B_i/B_0 . For large $\tan \beta$, however, the first two terms become larger when renormalized at the electroweak scale, as they are generated through radiative corrections involving bottom Yukawa couplings [34, 35]; there is still a cancellation of about $10^{-4} \tan^2 \beta$ in $(\mu_i/\mu_0 - B_i/B_0)$ and about $10^{-3} \tan^2 \beta$ in $(m_{L00}^2 - m_{Lii}^2)$. In the end, ϵ'_i can be as small as $10^{-2} \times \mathcal{O}(\mu_i/\mu_0) \approx 10^{-2} \times \mathcal{O}(B_i/B_0)$, but it is unlikely that ϵ'_i 's are even smaller than that.

A.1 Mass Matrices

Neutralino–Neutrino Mixing Because of $v_i \neq 0$ and $\mu_i \neq 0$, mass matrices of neutralinos and neutrinos are mixed up. In the gauge-eigenstate basis $\psi^0 = (\tilde{B}, \tilde{W}^0, \tilde{H}_u^0, \tilde{H}_d^0, \nu_i)^T$, the neutralino–neutrino mass matrix $\mathcal{L} \ni -\psi^{0T} M_{\tilde{N};7 \times 7} \psi^0/2$ becomes

$$M_{\tilde{N};7 \times 7} = \begin{pmatrix} M_{\tilde{N};4 \times 4} & m_{\tilde{N}}^T \\ m_{\tilde{N};3 \times 4} & m_{\text{ss}} \end{pmatrix}, \quad (\text{A.11})$$

where m_{ss} is the contribution from the see-saw mechanism involving right-handed neutrinos, and

$$M_{\tilde{N};4 \times 4} = \begin{pmatrix} M_1 & & -g'v_d/\sqrt{2} & g'v_u/\sqrt{2} \\ & M_2 & gv_d/\sqrt{2} & -gv_u/\sqrt{2} \\ -g'v_d/\sqrt{2} & gv_d/\sqrt{2} & 0 & -\mu_0 \\ g'v_u/\sqrt{2} & -gv_u/\sqrt{2} & -\mu_0 & 0 \end{pmatrix}, \quad (\text{A.12})$$

$$m_{\tilde{N};3 \times 4} = (-g'v_i/\sqrt{2}, gv_i/\sqrt{2}, 0, -\mu_i). \quad (\text{A.13})$$

Gauge-eigenstate basis ψ^0 and mass-eigenstate basis $\hat{\psi}^0$ are related by $\hat{\psi}^0 = N_{7 \times 7} \cdot \psi^0$, where a unitary matrix $N_{7 \times 7}$ makes $N_{7 \times 7}^* \cdot M_{\tilde{N};7 \times 7} \cdot N_{7 \times 7}^{-1}$ diagonal. Four mass eigenvalues are supposed to be around the electroweak scale or SUSY scale, and three others are very small. We will write these mass eigenstates as $\hat{\chi}_{1,2,3,4}^0$ and $\hat{\nu}_{1,2,3}$. Ignoring m_{ss} and anything that comes at the second order of bilinear R-parity violation, the diagonalization matrix is expressed in a form

$$N_{7 \times 7} \simeq \begin{pmatrix} N_{4 \times 4} & \\ & 1_{3 \times 3} \end{pmatrix} \begin{pmatrix} 1_{4 \times 4} & \xi^T \\ -\xi^* & 1_{3 \times 3} \end{pmatrix}, \quad (\text{A.14})$$

where $\xi \cdot M_{\tilde{N};4 \times 4} = m_{\tilde{N};3 \times 4}$ at this order, and $N_{4 \times 4}^* M_{\tilde{N};4 \times 4} N_{4 \times 4}^{-1}$ is diagonal. The suffixes of $\xi_{\alpha\beta}$ run over $\alpha \in \{\hat{\nu}_1, \hat{\nu}_2, \hat{\nu}_3\}$ and $\beta \in \{\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0\}$. We will keep the lower-right 3×3 part of $N_{7 \times 7}$ as $1_{3 \times 3}$, so that the three neutrino-like “massless” eigenstates $\hat{\nu}_{1,2,3}$ become $\nu_{e,\mu,\tau}$ approximately.

Assuming a little hierarchy between the electroweak symmetry breaking scale and the SUSY-breaking scale, i.e., $M_{Z,W} \ll M_{1,2}, \mu_0$, simple expressions for $\xi_{\alpha\beta}$ are obtained.

$$\xi_{\hat{\nu}_i \tilde{B}} \simeq \frac{M_Z}{M_1} \sin \theta_W \cos \beta \epsilon'_i, \quad (\text{A.15})$$

$$\xi_{\hat{\nu}_i \tilde{W}^0} \simeq -\frac{M_Z}{M_2} \cos \theta_W \cos \beta \epsilon'_i, \quad (\text{A.16})$$

$$\xi_{\hat{\nu}_i \tilde{H}_d^0} \simeq \frac{\mu_i + m_0 \tan \beta \epsilon'_i}{\mu_0}, \quad (\text{A.17})$$

$$\xi_{\hat{\nu}_i \tilde{H}_u^0} \simeq -\frac{m_0}{\mu_0} \epsilon'_i, \quad (\text{A.18})$$

where

$$m_0 \equiv \frac{(M_Z \cos \beta)^2 (M_1 \cos^2 \theta_W + M_2 \sin^2 \theta_W)}{M_1 M_2}. \quad (\text{A.19})$$

The tree-level contribution to the low-energy neutrino mass from the higgsino–neutrino mixing is given by $m_0 \epsilon'_i \epsilon'_j$ [33].

Chargino–Charged-Lepton Mixing Charginos and charged leptons also have a mixed mass matrix. In the gauge-eigenstate basis $\psi^+ = (\tilde{W}^+, \tilde{H}_u^+, e_i^c)^T$ and $\psi^- = (\tilde{W}^-, \tilde{H}_d^-, e_{Li})^T$, the mass matrix $M_{\tilde{C};5 \times 5}$ in $\mathcal{L} \ni -\psi^{-T} M_{\tilde{C};5 \times 5} \psi^+ + \text{h.c.}$ is given by

$$M_{\tilde{C};5 \times 5} = \begin{pmatrix} M_2 & gv_u & 0 \\ gv_d & \mu_0 & -y_i^e v_i \\ gv_i & \mu_i & y_i^e v_d \end{pmatrix}. \quad (\text{A.20})$$

Mass eigenstates $\hat{\psi}^+ = (\hat{\chi}_{1,2}^+, \hat{e}_{1,2,3}^c)$ and $\hat{\psi}^- = (\hat{\chi}_{1,2}^-, \hat{e}_{L1,2,3})$ are unitary transforms of the original gauge eigenstates;

$$\hat{\psi}^- = U \cdot \psi^-, \quad \hat{\psi}^+ = V \cdot \psi^+, \quad (\text{A.21})$$

where unitary matrices U and V make $U^*(M_{\tilde{C}} M_{\tilde{C}}^\dagger) U^T$ and $V(M_{\tilde{C}}^\dagger M_{\tilde{C}}) V^{-1}$ diagonal.

Under the same approximation as above, eigenvectors for the $\{e_L, \mu_L, \tau_L\}$ -like mass eigenstates $\hat{e}_{L1,2,3}$ are

$$U_{\hat{e}_{Li} \tilde{W}^-} \simeq \frac{\sqrt{2} M_W \cos \beta}{M_2} \epsilon'_i, \quad U_{\hat{e}_{Li} \tilde{H}_d^-} \simeq -\frac{\mu_i}{\mu_0}. \quad (\text{A.22})$$

Eigenvectors for the $\{e^c, \mu^c, \tau^c\}$ -like mass eigenstates $\hat{e}_{1,2,3}^c$ are

$$V_{\hat{e}_i^c \tilde{W}^+} \simeq \frac{\sqrt{2} M_W (M_2 \sin \beta + \mu_0 \cos \beta)}{M_2^2} \frac{m_i^e}{\mu_0} \epsilon'_i, \quad V_{\hat{e}_i^c \tilde{H}_u^+} \simeq -\frac{m_i^e}{\mu_0} \epsilon'_i. \quad (\text{A.23})$$

Charged Scalar Mixing There are eight complex scalar fields with +1 electric charge in the MSSM, namely, $H_u^+, H_d^{-*}, \tilde{e}_i^c, \tilde{e}_{Li}^*$, one of which is the longitudinal component of W^+ . It is convenient to adopt the $v_i = 0$ basis, (H'_d, L'_i) . The $v_i = 0$ basis and the bilinear basis are related by

$$\begin{pmatrix} H'_d \\ L'_i \end{pmatrix} \simeq \begin{pmatrix} 1 & v_i/v_d \\ -v_i/v_d & 1_{3 \times 3} \end{pmatrix} \begin{pmatrix} H_d \\ L_i \end{pmatrix}. \quad (\text{A.24})$$

In the new basis $\phi^+ = (H^+, H^{-'*}, \tilde{e}_i^c, \tilde{e}_{Li}^*)^T$, mass matrix in $\mathcal{L} \ni -\phi^{+\dagger} M_{C;8 \times 8} \phi^+$ is given by

$$M_{C;8 \times 8} = \left(\begin{array}{cc|cc} M_u^2 & B'_0 & m_q^e \mu'_q & B'_j \\ B'_0 & M_d^2 & m_q^e \mu'_q \tan \beta & M_{L0j}^2 \\ \hline m_p^e \mu'_p & m_p^e \mu'_p \tan \beta & M_{Epq}^2 & M_{\text{mix}pj}^2 \\ B'_i & M_{Li0}^2 & M_{\text{mix}iq}^2 & M_{Lij}^2 \end{array} \right), \quad (\text{A.25})$$

where

$$B'_0 \simeq B_0 + \frac{1}{2} M_W^2 \sin(2\beta), \quad (\text{A.26})$$

$$B'_i \simeq B_i - (v_i/v_d) B_0, \quad (\text{A.27})$$

$$\mu'_i \simeq \mu_i - (v_i/v_d) \mu_0 = \epsilon'_i \mu_0 \quad (\text{A.28})$$

(i.e., $\{B'_i, \mu'_i\}$ are $\{B_i, \mu_i\}$ in the $v_i = 0$ basis), and

$$M_u^2 \equiv m_{H_u}^2 - \frac{1}{2} M_Z^2 \cos(2\beta) + M_W^2 \cos^2 \beta + \mu_0^2 + \mu_i'^2, \quad (\text{A.29})$$

$$M_d^2 \equiv m_{L00}'^2 + \frac{1}{2} M_Z^2 \cos(2\beta) + M_W^2 \sin^2 \beta + \mu_0^2, \quad (\text{A.30})$$

$$M_{Lij}^2 \equiv m_{Lij}'^2 + \left(\frac{1}{2} M_Z^2 - M_W^2 \right) \cos(2\beta) \delta_{ij} + \mu'_i \mu'_j + (m_i^e)^2 \delta_{ij}, \quad (\text{A.31})$$

$$M_{Epq}^2 \equiv m_{Epq}^2 + (M_Z^2 - M_W^2) \cos(2\beta) \delta_{pq} + (m_p^e)^2 \delta_{pq}, \quad (\text{A.32})$$

$$M_{L0j}^2 \equiv m_{L0j}'^2 + \mu_0 \mu'_j, \quad (\text{A.33})$$

$$M_{\text{mix}iq}^2 \equiv m_{iq}^e (A - \mu_0 \tan \beta). \quad (\text{A.34})$$

Here $m_{L00}'^2$, $m_{Li0}'^2$, $m_{L0j}'^2$ and $m_{Lij}'^2$ are non-holomorphic SUSY-breaking mass-square parameters in the $v_i = 0$ basis. The eigenvector corresponding to the would-be Goldstone boson is \propto

$(v_u, -v_d, 0, 0, 0, 0, 0)$. After removing this mode, the mass matrix becomes

$$M_{C;7 \times 7} = \left(\begin{array}{c|cc} M_{H^\pm}^2 & m_q^e \mu'_q / \cos \beta & B'_i / \cos \beta \\ \hline m_p^e \mu'_p / \cos \beta & M_{E pq}^2 & M_{\text{mix } pj}^2 \\ B'_i / \cos \beta & M_{\text{mix } iq}^2 & M_{L ij}^2 \end{array} \right). \quad (\text{A.35})$$

In the limit that charged lepton masses m^e are ignored, both $M_{\text{mix } i\ell}^2$ and $m_\ell^e \mu'_\ell$ vanish, and hence \tilde{e}^c 's do not mix with the charged Higgs boson and left-handed charged sleptons.

A.2 Three Point Vertices

We present some of three point interactions of the MSSM with bilinear R-parity violation. Three point interactions involving one SUSY-particle-like mass eigenstate and two Standard-Model-particle like mass eigenstates are relevant to two-body decay of the visible-sector LSP, and to nucleon decay amplitudes.

Squark Yukawa Couplings Here is a list of all the three point couplings that involve a squark and two Standard-Model fermions. They are found in [47]. These interactions are combined with \mathcal{O}_3 (3.5), \mathcal{O}_7 (3.7), \mathcal{O}_0'' (3.9) to generate nucleon decay amplitudes.

The three point interactions involving charged leptons are

$$\begin{aligned} \Delta \mathcal{L} = & -(g V_{\hat{e}_k^c \tilde{W}^+} V_{ij}^{CKM}) \tilde{u}_{Li}^* (\bar{e}_k P_L d_j) \\ & + (y_j^d U_{\hat{e}_{Lk} \tilde{H}_d^-})^* V_{ij}^{CKM} \tilde{u}_{Li}^* (\bar{e}_k P_R d_j) + (y_i^u V_{\hat{e}_k^c \tilde{H}_u^+} V_{ij}^{CKM}) \tilde{u}_{Ri}^* (\bar{e}_k P_L d_j) + \text{h.c.}, \end{aligned} \quad (\text{A.36})$$

$$\begin{aligned} \Delta \mathcal{L} = & -(g U_{\hat{e}_{Lk} \tilde{W}^-}) V_{ji}^{CKM*} \tilde{d}_{Li}^* (\bar{e}_k^c P_L u_j) \\ & + (y_j^u V_{\hat{e}_k^c \tilde{H}_u^+} V_{ji}^{CKM})^* \tilde{d}_{Li}^* (\bar{e}_k^c P_R u_j) + (y_i^d U_{\hat{e}_{Lk} \tilde{H}_d^-}) V_{ji}^{CKM*} \tilde{d}_{Ri}^* (\bar{e}_k^c P_L u_j) + \text{h.c.} \end{aligned} \quad (\text{A.37})$$

Here, \tilde{u}_{Li} and $V_{ij}^{CKM} \tilde{d}_{Lj}$ are complex scalar fields in the chiral multiplets $(u_{Li}, V_{ij}^{CKM} d_{Lj})$, and \tilde{u}_R and \tilde{d}_R [resp. $\tilde{u}_R^* = \tilde{u}^c$ and $\tilde{d}_R^* = \tilde{d}^c$] in the anti-chiral multiplets \bar{U}^\dagger and \bar{D}^\dagger [resp. in the chiral multiplets \bar{U} and \bar{D}]. Four component notations $\bar{e} P_L d$ and $\bar{e} P_R d$ mean $\hat{e}^c d_L$ and $\overline{\hat{e}_L d^c}$, respectively, and $\bar{e}^c P_L u = \bar{u}^c P_L e$ and $\bar{e}^c P_R u = \bar{u}^c P_R e$ correspond to $\hat{e}_L u_L$ and $\overline{\hat{e}^c u^c}$, respectively.

Those involving neutrinos are

$$\Delta\mathcal{L} = -\sqrt{2} \left(\frac{g'}{6} \xi_{\hat{\nu}_k \tilde{B}} + \frac{g}{2} \xi_{\hat{\nu}_k \tilde{W}^0} \right) \tilde{u}_{Li}^* (\bar{\nu}_k P_L u_i) - (y_i^u \xi_{\hat{\nu}_k \tilde{H}_u^0})^* \tilde{u}_{Li}^* (\bar{\nu}_k P_R u_i) + \text{h.c.}, \quad (\text{A.38})$$

$$-\sqrt{2} \left(\frac{g'}{6} \xi_{\hat{\nu}_k \tilde{B}} - \frac{g}{2} \xi_{\hat{\nu}_k \tilde{W}^0} \right) \tilde{d}_{Li}^* (\bar{\nu}_k P_L d_i) - (y_i^d \xi_{\hat{\nu}_k \tilde{H}_d^0})^* \tilde{d}_{Li}^* (\bar{\nu}_k P_R d_i) + \text{h.c.}, \quad (\text{A.39})$$

$$-\sqrt{2} \left(-\frac{2}{3} g' \xi_{\hat{\nu}_k \tilde{B}} \right)^* \tilde{u}_{Ri}^* (\bar{\nu}_k P_R u_i) - (y_i^u \xi_{\hat{\nu}_k \tilde{H}_u^0}) \tilde{u}_{Ri}^* (\bar{\nu}_k P_L u_i) + \text{h.c.}, \quad (\text{A.40})$$

$$-\sqrt{2} \left(+\frac{1}{3} g' \xi_{\hat{\nu}_k \tilde{B}} \right)^* \tilde{d}_{Ri}^* (\bar{\nu}_k P_R d_i) - (y_i^d \xi_{\hat{\nu}_k \tilde{H}_d^0}) \tilde{d}_{Ri}^* (\bar{\nu}_k P_L d_i) + \text{h.c.} \quad (\text{A.41})$$

The four component spinor $\bar{\nu}_k \equiv (\hat{\nu}_k, \bar{\hat{\nu}}_k)$ is Majorana.

Neutralino Three-Point Vertices Here, we list three-point vertices involving the neutralino-like mass eigenstate $\hat{\chi}_1^0$ that allow its two-body decay to two Standard-Model fields.

$$\Delta\mathcal{L} = -\bar{\hat{\nu}}_k \bar{\sigma}^\mu Z_\mu \hat{\chi}_1^0 \times \frac{gZ}{2} \left(\xi_{\hat{\nu}_k \tilde{B}} N_{4 \times 4; 1\tilde{B}} + \xi_{\hat{\nu}_k \tilde{W}^0} N_{4 \times 4; 1\tilde{W}^0} + 2\xi_{\hat{\nu}_k \tilde{H}_u^0} N_{4 \times 4; 1\tilde{H}_u^0} \right)^* + \text{h.c.} \quad (\text{A.42})$$

$$-\bar{\hat{e}}_k^c \bar{\sigma}^\mu W_\mu^+ \hat{\chi}_1^0 \times \frac{g}{\sqrt{2}} \left(-\sqrt{2} V_{\hat{e}_k^c \tilde{W}^+} N_{1\tilde{W}^0}^* + V_{\hat{e}_k^c \tilde{H}_u^+} N_{1\tilde{H}_u^0}^* \right) + \text{h.c.}, \quad (\text{A.43})$$

$$-\bar{\hat{e}}_{Lk} \bar{\sigma}^\mu W_\mu^- \hat{\chi}_1^0 \times \frac{g}{\sqrt{2}} \left(\sqrt{2} U_{\hat{e}_{Lk} \tilde{W}^-} N_{1\tilde{W}^0}^* + U_{\hat{e}_{Lk} \tilde{H}_d^-} N_{1\tilde{H}_d^0}^* + \sum_\alpha \xi_{\hat{\nu}_k \alpha}^* N_{1\alpha}^* \right) + \text{h.c.} \quad (\text{A.44})$$

In the last line, α runs over $\{\tilde{B}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0\}$. Higgs–neutralino–neutrino three point vertices are omitted.

Stau Three Point Vertices When the left-right mixing is ignored, $\tilde{\tau}^c$ is a mass-eigenstate $\hat{\tilde{\tau}}^c$ itself. In this limit, R-parity violating vertices including stau, which can be used for the calculation of stau–vLSP decay, are given as

$$\Delta\mathcal{L} = (\sqrt{2} g' \xi_{\hat{\nu}_k \tilde{B}}^*) \tilde{\tau}^c (\bar{\nu}_k P_R \tau) \quad (k = e, \mu, \tau) \quad (\text{A.45})$$

$$+ (y_3^e \xi_{\hat{\nu}_k \tilde{H}_d^0}) \tilde{\tau}^c (\bar{\nu}_k P_L \tau) + (y_3^e U_{\hat{e}_{Lk} \tilde{H}_d^-}^*) \tilde{\tau}^c (\bar{\nu}_\tau P_L e_k) \quad (k \neq \tau) \quad (\text{A.46})$$

$$+ (y_3^e (U_{\hat{\tau}_L \tilde{H}_d^-}^* + \xi_{\hat{\nu}_\tau \tilde{H}_d^0})) \tilde{\tau}^c (\bar{\nu}_\tau P_L \tau) + \text{h.c.} \quad (\text{A.47})$$

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